Formulae (these are not given on the formula sheet)

The integrating factor for the differential equation $\frac{dy}{dx}+P(x)y=Q(x)$ is $I(x)=e^{\int P(x)dx}$.

To solve a first order separable differential equation, use the formula $I(x)y=\int I(x)Q(x)dx$.

Example 1 – no initial conditions

Solve $\frac{dy}{dx}+y=6e^{2x}$. Express your answer in the form $y=f(x)$.

Solution

Step one: rearrange into the standard form $\frac{dy}{dx}+P(x)y=Q(x)$.

On this occasion, it is already in this form, so we do not need to rearrange.

Step two: identify P(x) and Q(x)

$P(x)=1$ and $Q(x)=6e^{2x}$.

Step three: find the integrating factor using the formula $I(x)=e^{\int P(x)dx}$.

$$I(x)=e^{\int P(x)dx}$$
$$=e^{\int 1dx}=e^{x}$$

Step four: use the formula $I(x)y=\int I(x)Q(x)dx$ to find the general solution, and rearrange to make y the subject.

$$I(x)y=\int I(x)Q(x)\ dx$$
$$e^{x}y=\int e^{x}\cdot 6e^{2x}\ dx$$
$$e^{x}y=6\int e^{3x}\ dx$$
$$e^{x}y=6\cdot\frac{1}{3}e^{3x}+C$$
$$y=2e^{2x}+\frac{C}{e^{x}}$$

Sometimes the equation must be rearranged into the standard form before we can solve it.

Example 2 – requires rearranging

Solve $x\frac{dy}{dx}-2y-x^{3}\sin x=0$. Express your answer in the form $y=f(x)$.

Solution

Step one: rearrange into the standard form $\frac{dy}{dx}+P(x)y=Q(x)$.

$$x\frac{dy}{dx}-2y-x^{3}\sin x=0$$
$$x\frac{dy}{dx}-2y=x^{3}\sin x$$ (move x^3sinx to the other side)

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