Expressions and Formulae Unit

Surds and Indices

Simplifying Surds

Definition: a **surd** is a square root (or cube root etc.) which does not have an exact answer. e.g. $\sqrt{2} = 1\cdot414213562...$, so $\sqrt{2}$ is a surd. However $\sqrt{9} = 3$ and $\sqrt[3]{64} = 4$, so $\sqrt{9}$ and $\sqrt[3]{64}$ are not surds because they have an exact answer.

We can multiply and divide surds.

Facts

$$\sqrt{a} \times \sqrt{b} = \sqrt{ab} \qquad \frac{\sqrt{a}}{\sqrt{b}} = \sqrt{\frac{a}{b}} \qquad \sqrt{x} \times \sqrt{x} = \left(\sqrt{x}\right)^2 = x$$

Example 1

Simplify $3\sqrt{2} \times 5\sqrt{2}$

Solution

To simplify a surd, look for square numbers that are factors of the original number.

Examples 2

Express $\sqrt{48}$ and $\sqrt{98}$ in their simplest form.

Solution

You can only add or take away surds when the number underneath the surd sign is the same.

For example: $\sqrt{5}+\sqrt{3}$ is NOT $\sqrt{8}$. Instead the simplest answer is $\sqrt{5}+\sqrt{3}$ (i.e. the expression does not change), because no simplifying is possible.

Examples 3 – simplifying a surd followed by collecting like terms

Write as a single surd in its simplest form: $\sqrt{63}+\sqrt{7}-\sqrt{28}$ **.**

Solution

Rationalising the Denominator

For various mathematical reasons, it is not good to have a surd on a bottom of a fraction.

Definition: Rationalising the denominator means turning the surd at the bottom of the fraction into a whole number, whilst keeping the fraction the same.

The method is very simple: **multiply top and bottom of the fraction by the surd**.

Example 1

Express with a rational denominator: $\frac{4}{\sqrt{5}}$

Solution

Example 2

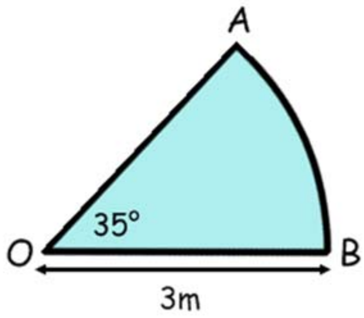
Express with a rational denominator: $\frac{1}{3\sqrt{2}}$

Solution

Example 2 – Sector area

Calculate the area of sector AOB in this diagram.

Solution



Note: units for sector area must always be squared units.

Volumes of Solids

You should know from National 4 how to calculate the volume of a **prism**. At National 5 level, you also need to be able to calculate the volume of a **pyramid**. Throughout this topic remember that:

- All volume questions must have answered in cubed units (e.g. m^3, cm^3, $inches^3$).
- You should always state your unrounded answer before rounding (see page 6).

Formula. This formula is **not** given on the National 5 Mathematics exam paper.

Volume of a Prism:

$$V = Ah$$

$$\text{Volume} = \text{Area of cross section} \times \text{Height}$$

Formula. This formula is given on the National 5 Mathematics exam paper.

Volume of a Pyramid:

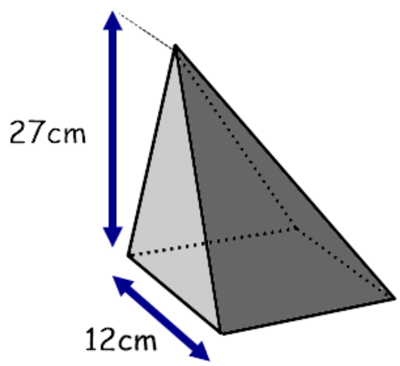
$$V = \frac{1}{3}Ah$$

$$\text{Volume} = \frac{1}{3}\text{Area of Base} \times \text{Height}$$

Example 1 – Pyramid

The diagram shows a pyramid with height 27cm and a square base with sides of length 12cm. Calculate the volume of the pyramid.

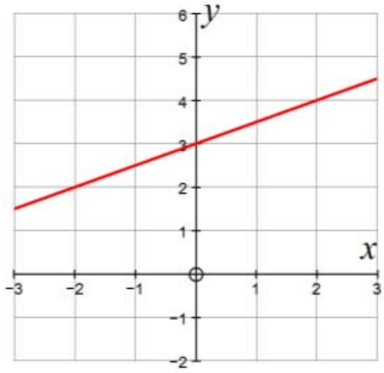
Solution



Relationships Unit

Straight Lines

The Equation of a Straight Line



The equation of any straight line is linked to the gradient of the line, and the *y*-intercept of the line:

- In the expressions and formulae unit, it is explained how to work out the **gradient** of a straight line. For example, it is shown that the gradient of the line on the right is ½ (see page 35).
- **Definition:** the ***y*-intercept** of a straight line is the number on the *y*-axis that the line passes through. For the line on the right, the *y*-intercept is 3.

Formula
The equation of any straight line can be written $y = mx + c$, where *m* is the gradient and *c* is the *y*-intercept of the line.

In everyday language, this means that:

- The gradient is "the number before *x*".
- The *y*-intercept is "the number that is not before *x*".

Examples

Write down the gradient and *y*-intercept of each straight line shown in the table:

Equation	Gradient	*y*-intercept
$y = 2x - 5$		
$y = 1{\cdot}5x + 4$		
$y = 8 - x$		
$y = 4 - 3x$		

Example 2

What is the equation of the straight line shown at the top of the page?

Solution

Another method for calculating the equation of a straight line from a diagram or sketch is shown on page 45.

To identify the gradient and *y*-intercept, the equation must begin ' $y = ...$ ' (that is, *y* must be the **subject**). If it does not, the equation must be rearranged to make *y* the subject.

Example 3

Calculate the gradient and *y*-intercept of the straight lines

(a) $3y = 6x - 9$ **(b)** $x + y = 5$ **(c)** $4y - 8x = 4$

Solution

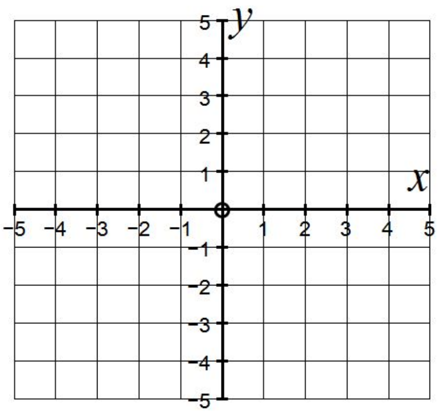
Drawing a Straight Line from its Equation

You need to know how to draw a line when given its equation. At National 4, you used a table of values. You can still do this. However, there is a quicker way involving a $y = mx + c$.

Example 1 – Drawing a straight line accurately

Draw the straight line $y = 2x - 5$.

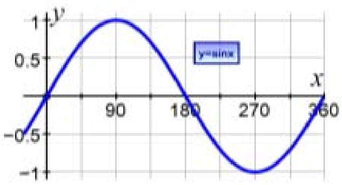
Solution



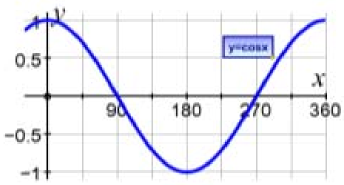
Trigonometry

Graphs of sin, cos and tan

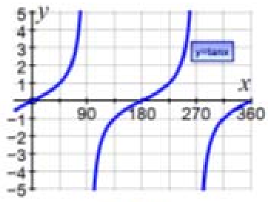
You should know what the graphs of sin x, cos x and tan x look like between 0° and 360°:



sin



cos



tan

Definition:

- the **frequency** of a sin or cos graph is how many times the graph repeats itself in 360°.
- The **frequency** of a tan graph is how many times it repeats itself in 180°.

In the equation of a sin, cos or tan graph, the frequency is the number multiplying *x*.

Definition: the **amplitude** is a measure of the 'height' of a sin or cos graph:

- the graphs above (with maximum 1 and minimum –1) both have an amplitude of 1.
- a sin or cos graph with a maximum of 8 and a minimum of –8 would have amplitude 8.

In the equation of a sin or cos graph, the amplitude is the number before sin or cos.

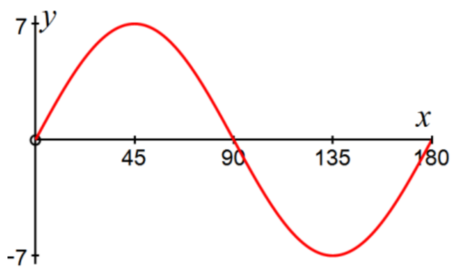
Definition: the **period** of a graph describes how many degrees it takes the graph to make one complete cycle. In the graphs above, sin x and cos x have a period of 360° and tan x has a period of 180°.

$$\text{Period of a sin or cos graph} = \frac{360^\circ}{\text{Frequency}} \qquad \text{Period of a tan graph} = \frac{180^\circ}{\text{Frequency}}$$

Equation	Frequency	Amplitude	Period
$y = \cos x$	1	1	360°
$y = 3\sin 4x$	4	3	90°
$y = 6\cos 2x$	2	6	180°
$y = 5\tan 2x$	2	5	90°

Example 1

The graph on the right has an equation of the form $y = a\sin bx$. State the values of *a* and *b*.



Solution