National 5
Applications of Mathematics
Revision Notes

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Use this booklet to practise working independently like you will have to in an exam.

- Get in the habit of turning to this booklet to refresh your memory.
- If you have forgotten how to do a method, **examples** are given.
- If you have forgotten what a word means, use the **index** (back pages) to look it up.

As you get closer to the exam, you should be aiming to use this booklet less and less.

This booklet is for:
- Students doing the National 5 Applications of Mathematics course.
- Students studying one or more of the National 5 Applications of Mathematics units: **Numeracy**, **Geometry and Measures** or **Managing Finance and Statistics**.

This booklet contains:
- The most important facts you need to memorise for National 5 Applications of Mathematics.
- Examples that take you through the most common **routine** questions in each topic.
- Definitions of the key words you need to know.

Use this booklet:
- To refresh your memory of the method you were taught in class when you are stuck on a homework question or a practice test question.
- To memorise key facts when revising for the exam.

The key to revising for a maths exam is to do questions, not to read notes. **As well as using this booklet, you should also:**
- Revise by working through exercises on topics you need more practice on – such as revision booklets, textbooks, websites, or other exercises suggested by your teacher.
- Work through practice tests.
- Ask your teacher when you come across a question you cannot answer.
- Use resources online (a link that can be scanned with a Smartphone is on the last page).

Contents

Formula Sheet

The following formulae are mentioned in these notes and are collected on this page for ease of reference.

Formulae that <u>are given</u> on the formula sheet in the exam (or in unit assessments)

Topic	**Formula(e)**	**Page Reference**
Pythagoras' Theorem	$a^2 + b^2 = c^2$	See page 53
Gradient	$\text{Gradient} = \dfrac{\text{Vertical height}}{\text{Horizontal distance}}$	See page 56
Circumference of a Circle	$C = \pi d$	See page 61
Area of a Circle	$A = \pi r^2$	See page 61
Volume of a prism	$V = Ah$	See page 65
Volume of a cylinder	$V = \pi r^2 h$	See page 65
Volume of a cone	$V = \dfrac{1}{3}\pi r^2 h$	See page 66
Volume of a sphere	$V = \dfrac{4}{3}\pi r^3$	See page 67
Standard deviation	$\sqrt{\dfrac{\sum (x-\overline{x})^2}{n-1}}$ or $\sqrt{\dfrac{\sum x^2 - \left(\sum x\right)^2 / n}{n-1}}$	See page 94

Formulae that are <u>not</u> given in the exam (or in unit assessments)

Topic	**Formula(e)**	**Page Reference**
Percentage increase and decrease	$\dfrac{\text{increase (or decrease)}}{\text{original amount}} \times 100$	See page 15
Percentage profit and loss	$\dfrac{\text{profit or loss}}{\text{expenditure}} \times 100$	See page 16
Perimeter of any shape	P = total of all outside lengths	See page 22
Area of a rectangle	$A = L \times B$	See page 22
Area of a triangle	$A = B \times H \div 2$	See page 22
Volume of a cuboid	$V = L \times B \times H$	See page 23
Speed, Distance, Time	$S = D \div T$ $\quad T = D \div S$ $\quad D = S \times T$	See page 23
Net Pay	Net Pay = Gross Pay – Total Deductions	See page 73
InterQuartile Range (IQR)	$\text{IQR} = \text{upper quartile} - \text{lower quartile}$ $(Q_3 - Q_1)$	See page 93

In the grid method, each number is split up according to the place value of its digits (e.g. 58 is split into 50 and 8; 238 is split into 200, 30 and 8) and then a mini 'tables square' grid is produced. All the answers in the grid are then added together.

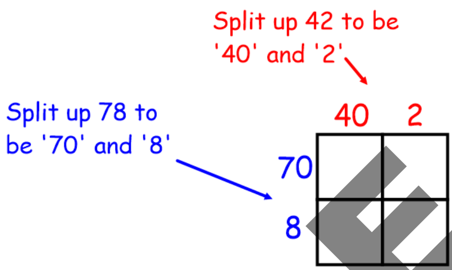
BASIC SKILL EXAMPLE 5: Multiplying two two-digit numbers

Multiply 78 × 42

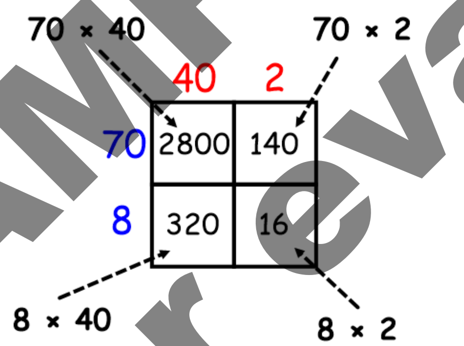
<u>**Solutions**</u>

Method A (Grid Method)

<u>Step 1:</u> construct a multiplication square with two numbers along the side and two numbers along the top.



<u>Step 2:</u> multiply the numbers in each row and column to obtain one number in each of the four smaller squares.



<u>Step 3:</u> add the four numbers to obtain the final answer.

2800 + 140 + 320 + 16 = <u>3276</u>

Method B (Long Multiplication)

<u>Step 1:</u> start doing a usual multiplication sum and do 78 × 2 using the usual method.

$$\begin{array}{r} 78 \\ \times\ 42 \\ \hline {}^{1}156 \end{array}$$

<u>Step 2:</u> the next line will be for 78 × 4. However, because the sum should really be 78 × 40, we write one zero in the units column.

$$\begin{array}{r} 78 \\ \times\ 42 \\ \hline {}^{1}156 \\ 0 \end{array}$$

<u>Step 3:</u> complete the sum 78 × 4 usual the usual method.

$$\begin{array}{r} 78 \\ \times\ 42 \\ \hline {}^{1}156 \\ 3{}^{3}120 \end{array}$$

<u>Step 4:</u> add the two answers to obtain the final answer.

156 + 3120 = <u>3276</u>

These methods can be extended to any multiplication sum, including multiplication of three- (or more)-digit numbers, or multiplication of decimals.

<u>Assessment Style Example 1: Multiplying a three-digit number and a two-digit number</u>

It costs £56 to cover one square metre of pathway with concrete. Calculate the cost to cover a path measuring 247m².

BASIC SKILL EXAMPLE 1: Finding the Percentage

Out of 1250 pupils, 475 get to school by bus. Express this as a percentage.

Solution

As a fraction, this is $\frac{475}{1250}$.

To change this to a percentage divide and then multiply by 100:

$475 \div 1250 \times 100 = \underline{38\%}$

Without a calculator, the calculation can be found using equivalent fractions. Multiply and divide the top and bottom by the same number to obtain the number 100 on the bottom of the fraction. The number on the top is then the percentage.

BASIC SKILL EXAMPLE 2: Finding the Percentage (non-calculator)

Darren baked 20 cakes. 13 of these cakes are carrot cakes.
Calculate the percentage of cakes that were carrot cakes.

Solution

The fraction of carrot cakes is $\frac{13}{20}$. We need to change this into a percentage. To obtain the number 100 on the bottom of the fraction we need to multiply both top and bottom by 5:

$\frac{13^{\times 5}}{20_{\times 5}} = \frac{65}{100}$, so the percentage is $\underline{65\%}$.

More difficult questions ask you to find the percentage increase or decrease. In these questions, you must always calculate the percentage of the **original** amount.

Formula: not given on the formula sheet in National 5 assessments

$$\text{Percentage increase/decrease} = \frac{\text{change}}{\text{original amount}} \times 100$$

BASIC SKILL EXAMPLE 3: Finding the Percentage Increase or Decrease

The temperature in an oven was 180°C. It increased to 207°C.
Calculate the percentage increase in temperature.

Solution

Step one: what is the increase? 207 – 180 = 27°C.

Step two: write as a fraction of the original amount:

Original amount was 180°C, so as a fraction this is $\frac{27}{180}$.

Step three: divide and multiply by 100 to change to a percentage:

$27 \div 180 \times 100 = \underline{15\%}$

You can only add and subtract fractions when the denominators are the same. When they are not, we must change the fractions into equivalent fractions with the same denominator.

BASIC SKILL EXAMPLE 7: Adding or subtracting fractions

Take away: $\frac{7}{8} - \frac{2}{3}$.

Solution

Step one: identify a common multiple of 8 and 3. 24

Step two: change both fractions to have 24 on the bottom: $\frac{21}{24}$ and $\frac{16}{24}$

Step three: take away: $\frac{21}{24} - \frac{16}{24} = \frac{5}{24}$

Exam questions often involve both adding and taking away.

Assessment Style Example 4 – adding and subtracting fractions

A shop sells T shirts in three sizes: small, medium and large.

- $\frac{3}{8}$ **of the T shirts are large.**
- $\frac{2}{5}$ **of the T shirts are medium.**
- **The rest are small.**

Calculate the fraction of T shirts that are small.

Solution

Step one: identify a common multiple of 8 and 5. This could be **40**.

Step two: change both fractions to have 40 as the denominator

$$\frac{3^{\times 5}}{8_{\times 5}} = \frac{15}{40} \quad \text{and} \quad \frac{2^{\times 8}}{5_{\times 8}} = \frac{16}{40}$$

Step three: add both fractions. $\frac{15}{40} + \frac{16}{40} = \frac{31}{40}$

Step four: subtract from one whole

$$1 - \frac{31}{40} = \frac{40}{40} - \frac{31}{40} = \underline{\underline{\frac{9}{40}}}$$

An **indirect proportion** (or **inverse proportion**) question is one in which when one set of numbers *increases*, the other set of numbers *decreases* at the same rate.

When two quantities are in indirect proportion, the **product** of the numbers is always the same (this means that when you multiply the numbers, the answer is always the same).

BASIC SKILL EXAMPLE 2: Indirect proportion

When 6 builders are employed, they can build a shed in 4 days.

If all builders work at the same rate, calculate how long it will take to build the same shed when 8 builders are employed.

Solution

When two variables are in indirect proportion, their product is always the same.

Step one: multiply to get the total number of days:

$4 \times 6 = 24$.

Step two: divide to share the total days out equally between 8 builders:

$24 \div 8 = 3$ so the job would take 3 days.

It is unlikely that an exam question would use the words 'indirect proportion'. Instead you would need to realise that indirect proportion is involved. You will know if you realise that one quantity *decreases* when the other quantity *increases*.

Assessment Style Example (adapted from SQA exam paper 2018: Paper 2 question 5a)

A container of sheep food will feed 350 sheep for 18 days.
The number of sheep increases by 100.
All sheep eat at the same rate.
Calculate how many days the same container will now last.

Solution

If the number of sheep increases, the number of days decreases. Therefore this is an inverse proportion question.

The number of sheep increases by 100, so there are now 450 sheep (350 + 100).

Step one: multiply to get the total number of days of feed:

$350 \times 18 \quad = 6300$

Step two: divide the total days of feed equally between 450 sheep:

$6300 \div 450 =$ 14 days

Part b: use your diagram to calculate the real-life lengths.

Fact: To find out a real-life length,
multiply the length on the page by the scale factor.

We measure the distance on the page from the 'start' point to the 'end' point, (indicated by a dotted line in the diagram above). If the diagram has been drawn perfectly, the dotted line should measure 8·35 cm.
The real-life distance is then 8·35 × 200 = 1670 metres.

Container Packing

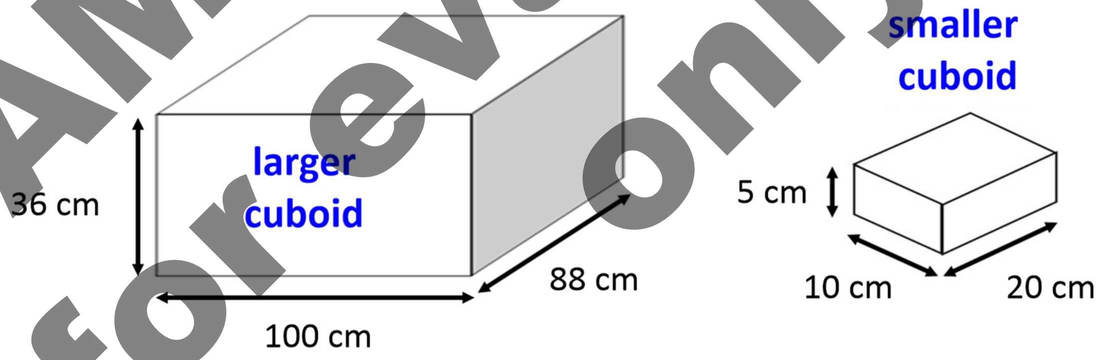
You need to be able to work out the best way of packing smaller three-dimensional objects inside larger containers. When doing so, we must bear several factors in mind:

- There are three dimensions. We will need to do one calculation for each dimension.
- The objects are solid, and so we cannot change the dimensions or have decimal answers.
- It is OK to have extra space left over inside the larger container. However, we want as little unused space as possible.
- Some objects may have to be stacked a certain way up so that they do not break.

To find out how many objects fit in, we need to divide the dimensions of the objects and the dimensions of the container. It is not possible to have a fraction of an object so if the answer is a decimal/fraction we must round **down** (never up) to the nearest whole number.

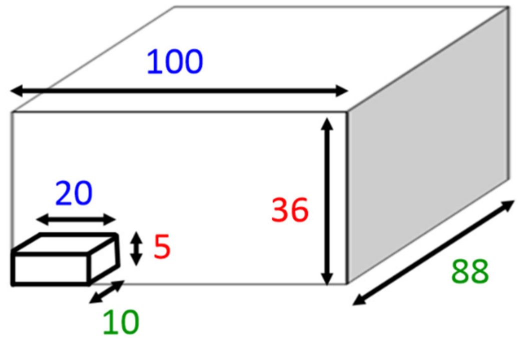
BASIC SKILL EXAMPLE 1: 3-d container packing where alignment is specified

Smaller cuboid boxes are being packed inside larger cuboid boxes.
The dimensions of each box are shown.



The boxes are all aligned as shown on the right (all lengths in centimetres).

Calculate how many smaller cuboid can fit inside the larger cuboid when the box is fully packed.



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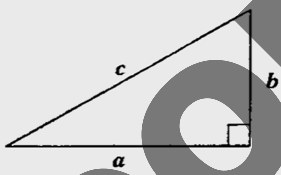
Geometry

Pythagoras' Theorem

At National 4 level you will have learnt that when you know the length of any two sides of a right-angled triangle you can use Pythagoras' Theorem (often just known as **Pythagoras**) to calculate the length of the third side without measuring.

Formula: given on the formula sheet in National 5 assessments

Theorem of Pythagoras:



$$a^2 + b^2 = c^2$$

There are three steps to any Pythagoras question:

Step One: square the length of the two given sides.

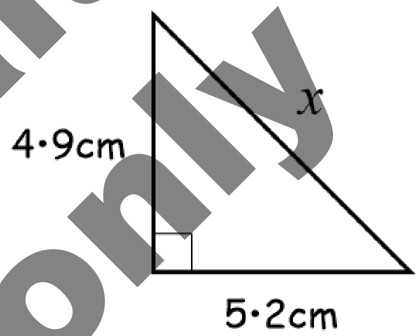
Step Two: either add or take away:

- To find the length of the longest side (hypotenuse), **add** the squared numbers.
- To find the length of a shorter side, **take away** the squared numbers.

Step Three: square root.

BASIC SKILL EXAMPLE 1: Pythagoras for the hypotenuse

Calculate the length of *x* in this triangle.



Solution

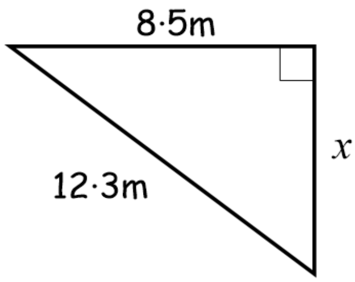
We are finding the length of *x*.

x is the hypotenuse, so we **add**:

$$x^2 = 4{\cdot}9^2 + 5{\cdot}2^2$$
$$x^2 = 51{\cdot}05$$
$$x = \sqrt{51{\cdot}05}$$
$$x = 7{\cdot}1449....$$
$$x = \underline{7{\cdot}1\text{cm}}$$

BASIC SKILL EXAMPLE 2: Pythagoras for a shorter side

Calculate the length of *x* in this triangle.



Solution

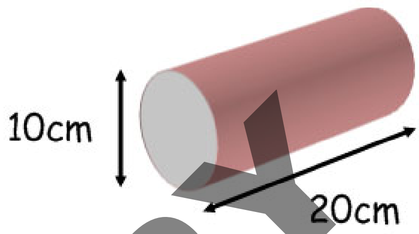
We are finding the length of *x*.

x is a smaller side, so we take away.

$$x^2 = 12{\cdot}3^2 - 8{\cdot}5^2$$
$$x^2 = 79{\cdot}04$$
$$x = \sqrt{79{\cdot}04}$$
$$x = 8{\cdot}8904....$$
$$x = \underline{8{\cdot}9\text{cm}}$$

BASIC SKILL EXAMPLE 2: Volume of a cylinder

Calculate the volume of this cylinder.



Solution

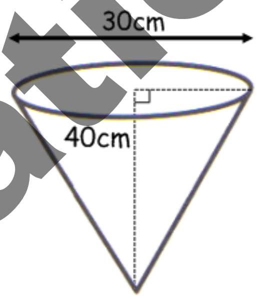
Diameter is 10 cm, so radius is 5 cm.

$$V = \pi r^2 h$$
$$= \pi \times 5^2 \times 20 \quad (\text{ or } \pi \times 5 \times 5 \times 20\ )$$
$$= 1570 \cdot 796327....$$
$$= \underline{1570 \cdot 8 \text{ cm}^3} \ (1 \text{ d.p.})$$

In the cone formula, the 'height' refers to the perpendicular height (the one that goes straight up) and not any sloping heights.

BASIC SKILL EXAMPLE 3: Volume of a cone

Calculate the volume of this cone.



Solution

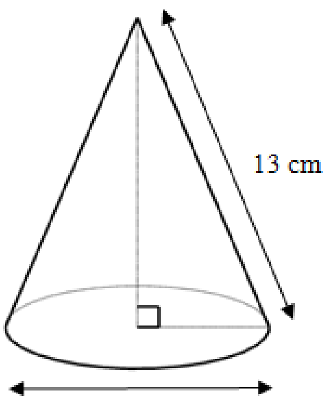
Diameter is 30 cm, so radius is 15 cm.

$$V = \frac{1}{3} \pi r^2 h$$
$$= \pi \times 15^2 \times 40 \div 3 \quad (\text{ or } 1 \div 3 \times \pi \times 15^2 \times 40\ )$$
$$= 9424 \cdot 777961....$$
$$= \underline{9424 \cdot 8 \text{ cm}^3} \ (1 \text{ d.p.})$$

If a sloping height is given rather than the perpendicular height, Pythagoras must be used to obtain the perpendicular height.

Assessment Style Example 1

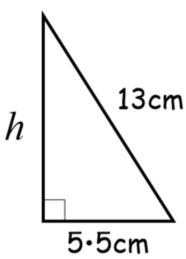
Metal parts for a machine are made in the shape of a cone, with diameter 11 cm and slant height 13 cm, as shown in the diagram.
There are 16 litres of (melted) metal. Calculate how many complete metal parts can be made.



Solution

The radius of the cone is 5·5 cm (half of 11 cm).

The radius, slant height and perpendicular height form a right-angled triangle as shown in the diagram below, in which the perpendicular height is labelled *h*.



We find *h* using Pythagoras:

$$h^2 = 13^2 - 5 \cdot 5^2$$
$$= 138 \cdot 75$$
$$h = \sqrt{138 \cdot 75}$$
$$= 11 \cdot 779...$$
$$= 11 \cdot 8 \text{ cm } (1 \text{ d.p.})$$

(continued on the next page)

Assessment Style Example 1 – with overtime

Rachel Frost works as a radiographer. Rachel works a basic 40-hour week (Monday – Friday) with a basic rate of £14·50 per hour.

- **Overtime on weekdays is paid time and a half.**
- **Overtime on weekends is at paid double time instead.**

In a particular week Rachel worked a total of 48 hours on weekdays, plus an additional 5 hours overtime on Sunday. Her total deductions for that week are £189·80.

Calculate Rachel's net pay for that week.

Solution

Rachel works 48-hours on weekdays, and her basic hours are 40 hours, so she worked 8 hours overtime.

Basic pay: £14·50 × 40 = £580
Weekday overtime (time-and-a-half): £14·50 × 8 × 1·5 = £174
Sunday pay (double time): £14·50 × 5 × 2 = £145

Gross pay: £580 + £174 + £145 = £899

Net pay: £899 – £189·80 = £709·20

Assessment Style Example 2 – with commission

Hamish Robertson works selling kitchens. He is paid 4·5% commission on total sales over £30 000.

During one particular month:

- **Hamish's basic pay is £1850 and his overtime pay is £124.**
- **His sales are £95 600.**
- **He pays 3% of his gross salary into his pension.**
- **He pays £575·30 in income tax and £245·08 in national insurance.**

Calculate Hamish's net pay for that month.

Solution

Gross pay:

Sales on which commission is payable = 95600 – 30 000 = £65 600.

Commission = 4·5% of £65 600 = 0·045 × 65600 = £2953

Gross salary = £1850 + £124 + £2953 = £4927

Deductions:

Pension = 3% of £4927 = 0·03 × 4927 = £147·81

Total Deductions = £575·30 + £245·08 + £147·81 = £968·19

Net Pay = Gross Pay – Total Deductions = £4927 – £968·19 = £3958·81

BASIC SKILL EXAMPLE 2: best deal

Mohammed is planning a group holiday to Cyprus. He has a choice of three packages:

Package A	**Package B**	**Package C**
7-night holiday: £898 per person. **Special Offer:** When 4 people book, a fifth person can go for free.	£96 per person per night.	10-person package holiday: £1200 per night. **Special Offer:** 22% discount on total price.

Mohammed wants to book a 7-night holiday for ten adults.
Determine which package is the cheapest option.

Solution

Package A: The special offer means that only 8 people out of 10 need to pay. We calculate the cost for 8 people paying £898 each.

Total price = £898 × 8 = £7184.

Package B: We calculate the total cost of £96 per person per night for 10 people and 7 nights.

96 × 10 × 7 = £6720.

Package C: The total price for 7 nights is £1200 × 7 = £8400. We now calculate the 22% discount on this price:

0·22 × 8400 = £1848
8400 – 1848 = £6552.

The *decision* is that Package C is the cheapest (because 6552 < 6720 and 7184).

Currency Exchange

When converting from one currency to another, an **exchange rate** is used. The exchange rate explains how many units of one currency you get for another. For example, the exchange rate for pounds sterling (£) into Euros (€) might be £1 = €1·27 *(for every one pound you exchange, you get 1·27 Euros in return)*.

In the UK, exchange rates are usually expressed in terms of pounds (i.e. £1 = _____). However, for people in other countries the exchange rate is likely to be expressed in terms of their own currency. For example, in France the exchange rate above would be likely to be expressed as €1 = £0·79 *(for every one Euro you exchange, you get 79 pence in return)*.

Exchange rates change regularly from day to day or even hourly depending on global events.

To do calculations with currencies we must either multiply or divide by the exchange rate. Which operation we choose depends on which way around we are converting.

Another way to purchase goods is to pay via **hire purchase** (HP) or a **payment plan**. This involves a one-off payment (the **deposit**) followed by paying the balance in **instalments**. Paying via HP makes the individual payments lower, but means that the total amount paid will be higher.

BASIC SKILL EXAMPLE 3: payment plan

The Liu family are going to purchase a car using a payment plan. Its cash price is £8400.

- **The total payment plan price is 15% more than the cash price.**
- **They will pay a deposit of $\frac{1}{3}$ of the payment plan price followed by 20 equal monthly instalments.**

Calculate the price of each monthly instalment.

Solution

Payment plan price = £8400 + 15% = 1·15 × 8400 = £9660.

Deposit = $\frac{1}{3}$ of payment plan price = 9660 ÷ 3 = £3220 , leaving 9660 – 3220 = £6440 to pay.

There are 20 instalments. Price of each equal instalment = 6440 ÷ 20 = £322.

When you borrow money on a **credit card** (or a store card), you do not have to pay it all back at once. Instead, you can choose how much you pay back and when. You can pay the entire balance off at once if you want, but you can pay a lot less if you want to so long as you pay at least the minimum payment set down by the company.

Definition: the **balance owed** on a credit card statement is how much money you owe the company at the current date.
Definition: the **Annual Percentage Rate (APR)** is the interest rate that you pay on your balance each year. By law, the APR must be stated for all credit cards, store cards and loans.

The credit card provider decides the **minimum payment** that you have to pay each month. This is often expressed as a percentage of the balance owed. Sometimes it may be expressed in a phrase such as '3% of the balance owed or £5, whichever is greater'.

BASIC SKILL EXAMPLE 4: credit cards and APR

Peri takes out a credit card with an APR of 35·96%.
She uses the credit card to buy goods costing £1400 and makes no repayments.
After one year, Peri must make a minimum repayment of "4% of the balance owed or £50, whichever is greater". Calculate Peri's minimum payment.

Solution

(a) The interest for one year is 35·96% of £1400 = 0·3596 × 1400 = £503·44
There are no repayments, so the only change in the balance is to add on the interest.
The balance owed is 1400 + 503·44 = £1903·44.

(b) 4% of the balance owed = 4% of £1903·44
= 0·04 × 1903·44 = 76·1376….
= £76·14 (must use two decimal places for money)

This is bigger than £50, so the minimum payment is £76·14

BASIC SKILL EXAMPLE 2: Drawing a Box Plot

Construct a boxplot for the following data about shoe sizes:

2 3 4 4 4 5 5 6 7 8 9

Solution

Step One: write down the lowest and highest values.

Lowest = 2. Highest = 9

Step Two: calculate the median. This list is already in order.

2 3 4 4 4 5 5 6 7 8 9

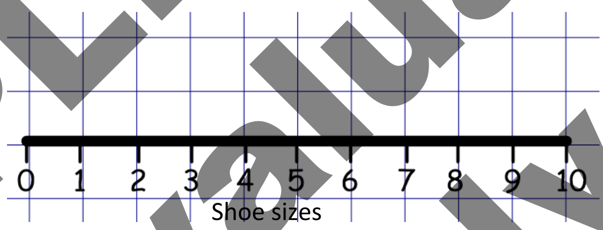
The median is 5.

Step Three: calculate the upper and lower quartiles using the method from the previous example.

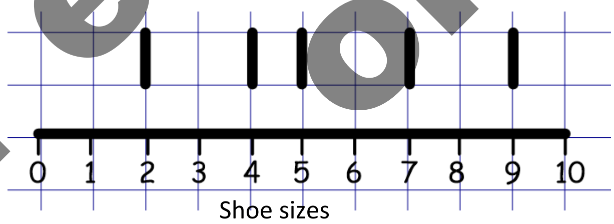
2 3 4 4 4 5 5 6 7 8 9

The lower quartile is 4 and the upper quartile is 7.

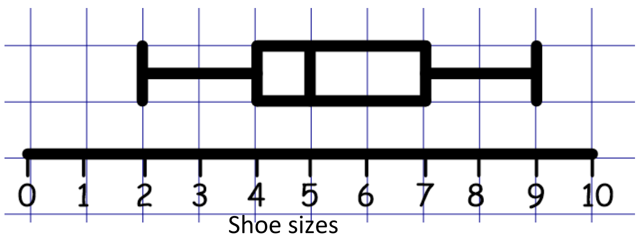
Step Four: draw and label a horizontal axis, including an overall label ("shoe sizes").



Step Five: draw five vertical lines corresponding to the five numbers calculated in steps 1-3. (the lowest, lower quartile, median, upper quartile and highest).



Step Six: join the middle three lines together to create a rectangle, and join the end points to create the full box plot shape.



Formula: given on the formula sheet in National 5 assessments

$$\text{standard deviation} = \sqrt{\frac{\sum (x-\overline{x})^2}{n-1}} \quad \text{or} \quad \sqrt{\frac{\sum x^2 - (\sum x)^2/n}{n-1}}$$

The following symbols are used in these formulae:

- n stands for 'how many numbers are in the list'
- $\overline{x}$ stands for 'the mean' ($\overline{x}$ is read out loud as 'x bar')
- Σ means "add together" (Σ is **sigma**, the Greek capital 'S')

You only need to know how to use one of these formulae. In general, it is more helpful to just know the method rather than memorising the formula. The following two examples show how the same question is done using each method.

BASIC SKILL EXAMPLE 1a: Standard Deviation using the formula $s = \sqrt{\frac{\sum (x-\overline{x})^2}{n-1}}$

Calculate the mean and standard deviation of this data set: 1 0 7 4 3

Solution

Step 1: Calculate the Mean.
There are five numbers, so $n = 5$.

Mean: $\frac{1+0+7+4+3}{5} = \frac{15}{5} = 3$, so the mean is 3.

Step 2: Draw up a table with column headings x, $x-\overline{x}$ and $(x-\overline{x})^2$.

x	$x-\overline{x}$	$(x-\overline{x})^2$
1		
0		
7		
4		
3		

Step 3: Complete the table, remembering that $\overline{x}$ = the mean = 3.

- *In the middle column, take away the mean from each number in the left-hand column.*
- *In the right-hand column, square each number in the middle column.*

x	$x-\overline{x}$	$(x-\overline{x})^2$
1	–2	4
0	–3	9
7	4	16
4	1	1
3	0	0
	TOTAL	**30**

Step 4: find the total of the final column
In this example, $\sum (x-\overline{x})^2 = 30$.

Step 5: use the formula, remembering that n = 5 because there were five numbers.

(continued on next page)

Index of Key Words

All information in this revision guide has been prepared in best faith, with thorough reference to the documents provided by the SQA, including the course arrangements, course and unit support notes, exam specification, specimen question paper and unit assessments.

These notes will be updated as and when new information becomes available.

We try our hardest to ensure these notes are accurate, but despite our best efforts, mistakes sometimes appear. If you discover any mistakes in these notes, please email us at david@dynamicmaths.co.uk.
An updated copy of the notes will be provided free of charge!
We would like to hear any suggestions you may have for improving our notes.

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