National 4 Mathematics
Revision Notes

Last updated January 2019

Use this booklet to practise working independently like you will have to in course assessments and the Added Value Unit (AVU).

- Get in the habit of turning to this booklet to refresh your memory.
- If you have forgotten how to do a method, **examples** are given.
- If you have forgotten what a word means, use the **index** (back page) to look it up.

As you get closer to the final test, you should aim to use this booklet less and less.

This booklet is for:

- Students doing the National 4 Mathematics course.
- Students studying one or more of the National 4 mathematics units: **Numeracy**, **Expressions and Formulae** or **Relationships.**

This booklet contains:

- The most important facts you need to memorise for National 4 Mathematics.
- Examples that take you through the most common **routine** questions in each topic.
- Definitions of the key words you need to know.

Use this booklet:

- To refresh your memory of the method you were taught in class when you are stuck on a homework question or a practice test question.
- To memorise key facts when revising for assessments and the Added Value Unit.

The key to revising for a maths test is to do questions, not to read notes. **As well as using this booklet, you should also:**

- Revise by working through exercises on topics you need more practice on – such as revision booklets, textbooks, websites, or exercises suggested by your teacher.
- Work through practice tests.
- Ask your teacher when you come across a question you cannot answer.
- Use resources online (a link that can be scanned with a SmartPhone is on the last page).

Contents

All information in this revision guide has been prepared in best faith, with thorough reference to the documents provided by the SQA, including the course arrangements, course and unit support notes, exam specification, specimen question paper and unit assessments.

These notes will be updated as and when new information becomes available.

We try our hardest to ensure these notes are accurate, but despite our best efforts, mistakes sometimes appear. If you discover any mistakes in these notes, please email us at david@dynamicmaths.co.uk.
An updated copy of the notes will be provided free of charge!
We would like to hear any suggestions you may have for improving our notes.

This version is version 2.0: published December 2018

Previous versions:
Version 1.6: July 2015
Version 1.5: October 2014
Version 1.4: April 2014
Version 1.3: January 2014
Version 1.2: July 2013
Versions 1.1 and 1.0. May 2013.

With grateful thanks to **Arthur McLaughlin** and **John Stobo** for proof reading.

Formula Sheet

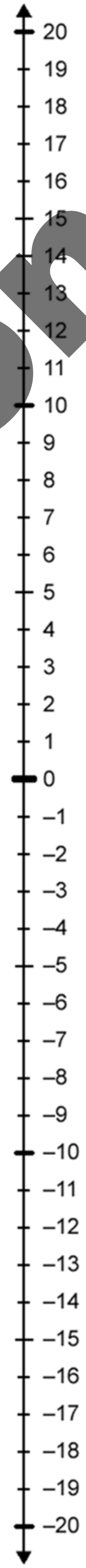
The following formulae are mentioned in these notes and are collected on this page for ease of reference.

Formulae that <u>are given</u> on the formula sheet in the Added Value Unit (or in unit assessments)

Topic	Formula(e)	Page Reference
Circumference of a circle	$C = \pi d$	See page 39
Area of a circle	$A = \pi r^2$	See page 39
Curved surface area of a cylinder	$A = 2\pi rh$	See page 47
Volume of a cylinder	$V = \pi r^2 h$	See page 44
Volume of a prism	$V = Ah$	See page 43
Gradient	$\text{Gradient} = \frac{\text{Vertical height}}{\text{Horizontal distance}}$	See page 37
Pythagoras' Theorem	$a^2 + b^2 = c^2$	See page 64
Trigonometry in a right-angled triangle	$\sin x° = \frac{\text{Opp}}{\text{Hyp}}$ $\cos x° = \frac{\text{Adj}}{\text{Hyp}}$ $\tan x° = \frac{\text{Opp}}{\text{Adj}}$	See page 66

Formulae that are <u>not</u> given in the Added Value Unit (or in unit assessments)

Topic	Formula(e)	Page Reference
Percentage increase and decrease	$\frac{\text{change}}{\text{original amount}} \times 100$	See page 17
Area of a rectangle	$A = LB$	See page 19
Area of a square	$A = L^2$	See page 19
Area of a triangle	$A = \frac{BH}{2}$	See page 19
Volume of a cuboid	$V = LBH$	See page 19
Speed, Distance, Time	$S = \frac{D}{T}$ $T = \frac{D}{S}$ $D = ST$	See page 22
Range	$\text{Range} = \text{Highest} - \text{Lowest}$	See page 49
Mean	$\text{Mean} = \frac{\text{Total}}{\text{How many}}$	See page 50
Equation of a straight line	$y = mx + c$	See page 59



Example 2 – adding and taking away

–6 + 9 = start at –6 and move up 9. Answer: **3**
5 – 7 = start at 5 and move down 7**.** Answer: **–2**
(–2) – 8 = start at –2 and move down 8. Answer: **–10**

Adding a negative number is the same as taking away. When an addition and a subtraction sign are written next to each other, you can "ignore" the addition sign.

Example 3– adding a negative

2 + (–6) = 2 – 6 = start at 2 and move down 6. Answer: **–4**
(–1) + (–7) = (–1) – 7 = start at –1 and move down 7. Answer: **–8**

Taking away a negative number becomes an addition. When two negative signs are written next to each other without a number in between, they become an add sign.

This can be thought of as "**taking away a negative becomes an add**"

Example 4 – taking away a negative

5 – (–2) = 5 + 2 = **7**
(–7) – (–2) = (–7) + 2 = start at –7 and move up 2. Answer: **–5**

Multiplying and Dividing Negative Numbers (Integers)

Multiplying and dividing integers have *completely different* rules to adding and taking away. To multiply and divide, you do the sum normally (as if there were no negative signs there), and then you decide whether your answer needs to be negative or positive.

When multiplying and dividing:

- If none of the numbers are negative, then the answer is **positive**.
- If one of the numbers is negative, then the answer is **negative**.
- If two of the numbers are negative, then the answer is **positive**.
- If three of the numbers are negative, then the answer is **negative**.

and so on…

In short, the rules are:

+ multiplied by **+** gives you **+**	**+** divided by **+** gives you **+**
– multiplied by **+** gives you **–**	**–** divided by **+** gives you **–**
+ multiplied by **–** gives you **–**	**+** divided by **–** gives you **–**
– multiplied by **–** gives you **+**	**–** divided by **–** gives you **+**

Example 1 – multiplication

(–5) × 4 = –20
60 × (–2) = –120

*If **one** of the numbers is negative, then the answer is **negative**.*

(–3) × (–10) = +30 (or just 30)
(–2) × 3 × (–4) = 24

*If **two** of the numbers are negative, then the answer is **positive**.*

(–2) × (– 3) × (–4) = –24

*If **three** of the numbers are negative, then the answer is **negative**.*

Graphs, Charts and Tables

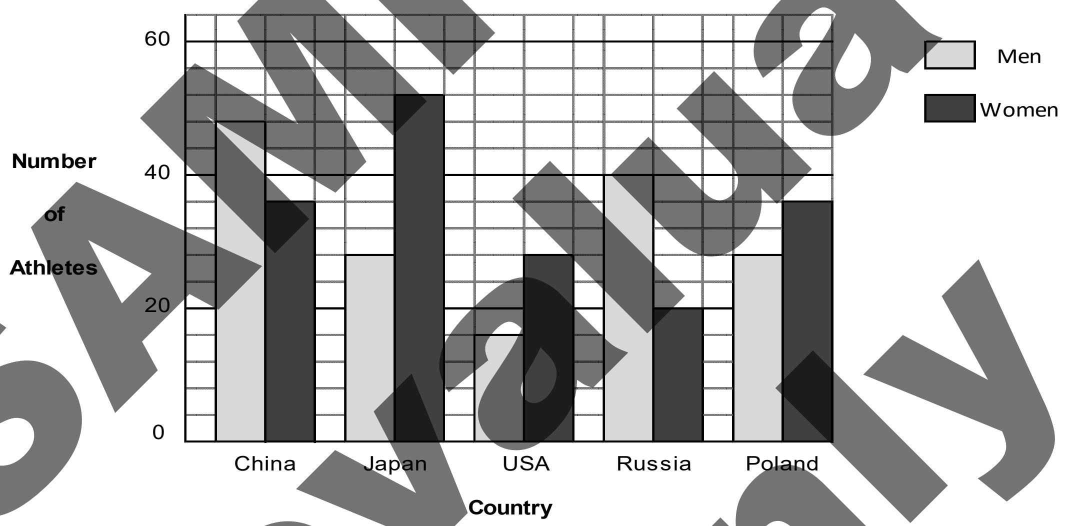
Interpreting and Comparing Graphs

To pass the National 4 numeracy unit, you need to be able to obtain information from at least two different types of diagram. These could include any sort of graph, chart or table – including a frequency table, a table of information, a bar chart, line graph, pie chart (see page 25), stem and leaf diagram (see page 26) or scatter graph (see page 73).

For the Added Value Unit you may be required to compare information and calculate differences from a graph or graphs. This is most likely to involve either a **bar graph** or a **pie chart**, though it could involve other graphs too.

Example 1 – Bar Chart, sample Added Value Unit question

The graph shows the number of athletes from five countries taking part in an international Sports Tournament.



(a) $\frac{2}{3}$ **of the male athletes from China were swimmers. Calculate the number of swimmers.**

(b) How many more athletes were from Japan compared to the USA?

Solution

(a) The graph tells us that there were 48 male athletes from China in total.

To find $\frac{2}{3}$ we divide by 3 and multiply by 2:

48 ÷ 3 × 2 = 32 athletes.

(b) The graph shows that there were 28 men and 52 women from Japan. This is a total of 80 athletes.

The graph also shows that there were 16 men and 28 women from the USA. This is a total of 44 athletes.

80 – 44 = 36, so there were 36 more athletes from Japan compared to the USA.

Ratio and Proportion

A **rate** is a way of comparing numbers when the figures are different. A rate is often expressed using the word 'per' which means 'for each one':

- Texts per month (how many texts did you send in one month?)
- Miles per hour (how many miles did you travel in one hour?)
- Pence per kilogram (how many pence does one kilogram cost?)

To calculate a rate, we divide. Order is important: the word before *per* is divided by the word after *per*. e.g. for pence per kilogram, our sum is pence ÷ kilograms.

Example

A car travels 300 miles on 20 gallons of fuel. Calculate the rate in miles per gallon.

Solution

For miles per gallon, we do the sum miles ÷ gallons: 300 ÷ 20 = 15.
Answer: 15 miles per gallon.

Ratio

Another way to describe the proportions in which quantities are split up is with **ratio**. Ratios consist of numbers separated by a colon symbol e.g., 2:3, 4:1, 3:2:4.

For example, it might be said that a particular shade of purple paint is made by mixing red paint and blue paint in the ratio 4:5. This means that for every 4 litres (or spoonfuls, tins, gallons…) of red paint, you must add 5 litres (or spoonfuls, tins, gallons…) of blue paint to get the correct shade of purple.

Example

Claire and David share £4500 in the ratio 4:5. Calculate the amount of money that they each get.

Solution

Claire gets 4 shares of the money, David gets 5 shares.
This means that there are **9 shares** in total.

Claire gets 4 out of 9 shares:

$\frac{4}{9}$ of £4500

$= 4500 \div 9 \times 4$

$= £2000$

David gets 5 out of 9 shares:

$\frac{5}{9}$ of £4500

$= 4500 \div 9 \times 5$

$= £2500$

Therefore, Claire gets <u>£2000</u> and David gets <u>£2500</u>.

Direct Proportion

A direct proportion question is one where you must use the fact that numbers change at the same rate. The method for one of these questions is usually to find the cost for *one* first.

Example 1

John hires a car for 4 days. It costs him £90.
His friend Sam hires the same car or 7 days. Calculate the cost Sam will pay.

Solution

Step One – How much does it cost to hire the car for one day?
Divide by 4: 90 ÷ 4 = 22·5, so it costs £22·50 per day.

Step Two – How much does it cost to hire the car for seven days?
Multiply by 7: 22·50 × 7 = £157·50.

An alternative to consider this problem is by considering equivalent ratios (by dividing or multiplying both numbers to create a new, equivalent ratio). A table usually helps to lay these calculations out:

	Days	:	Pounds	
÷ 4	4	:	90	÷ 4
× 7	1	:	22·5	× 7
	7	:	157·5	

The last entry in the final row shows us the answer is that 7 days hire costs £157·50 (remembering units and two decimal places for money)

Example 2

Jo is an electrician. She charges customers £27 for every 15 minutes she has to work. Calculate the cost for a job lasting 3 hours.

Solution

Step One – How much does it cost for one hour?
15 minutes cost £27. There are four lots of 15 minutes in an hour,
So for one hour it costs 27 × 4 = £108

Step Two – How much does it cost for three hours?
Multiply: 108 × 3 = £324

Using the alternative table approach (from Example 1) gives us a way of considering the mixture of units in the question. The right-hand column can be expressed in either minutes or hours, but either way we get the same answer of £324.

	Minutes	:	Pounds	
× 4	15	:	27	× 4
× 3	60	:	108	× 3
	180	:	324	

	Hours	:	Pounds	
× 4	0·25	:	27	× 4
× 3	1	:	108	× 3
	3	:	324	

Expressions and Formulae Unit

Algebra

Using a Formula

Read the question carefully and substitute in the numbers you are given.

Definition: Evaluate means "do the sum"

Example 1

Evaluate 2*a* – 3*c* when *a* = 12 and *c* = 1·5.

Solution

$2a - 3c$

$= 2 \times 12 - 3 \times 1 \cdot 5$

$= 24 - 4 \cdot 5$

$= \underline{19 \cdot 5}$

Example 2

Evaluate $3x^2$ when *x* = 5.

Solution

$3x^2$

$= 3 \times 5^2$

$= 3 \times 25$ (NOT 15^2 as squaring comes before multiplying in BODMAS)

$= \underline{75}$

Example 3

$S = 3bc - a$. Evaluate *S* when *a* = 10, *b* = 2 and *c* = 7.

Solution

$S = 3bc - a$

$= 3 \times 2 \times 7 - 10$

$= 42 - 10$

$= \underline{32}$

You would also be expected to understand a formula in a real-life situation, where the letters would be explained to you.

Example 4 – real life situation

The cost of hiring a car is given by the formula $C = 25d + 2p$, where *C* is the cost of hiring the car, *d* is the number of days hired for, and *p* is the number of litres of petrol used.

Calculate the cost of hiring the car for 3 days and using 40 litres of petrol.

Examples 2

Multiply out the brackets and simplify:

$4(m+5)-18$ $4(x+5)+3(x-2)$

Solution

$4(m+5)-18$
$=4m+20-18$
$=\underline{4m+2}$

$4(x+5)+3(x-2)$
$=4x+20+3x-6$
$=\underline{7x+14}$

Be careful: the only numbers (or letters) that you multiply by are the ones that are directly in front of the bracket. Anything else that not inside the bracket should remain unchanged until you start simplifying.

Examples 3

Multiply out the brackets and simplify:

$4+7(a+2)$ $2x+3(x+1)$

Solution

$4+7(a+2)$
$=4+7a+14$ (**NOT** $11(a+2)$)
$=\underline{18+7a}$ (or $7a+18$)

$2x+3(x+1)$
$=2x+3x+3$ (**NOT** $5x(x+1)$)
$=\underline{5x+3}$ (or $3+5x$)

Factorising

Definition: Factorise means "put the brackets back in". It is the reverse process to multiplying out the brackets.

Examples 1

Factorise $6a+9b$ **Factorise $15x+25y$**

Solution

The highest common factor is 3
Write 3 in front of the brackets
3( )

The highest common factor is 5
Write 5 in front of the brackets
5( )

Work out what goes inside the brackets (it may help to think of dividing)

Answer: $3(2a+3b)$ **Answer:** $5(3x+5y)$

You always need to take the ***largest possible*** number (and/or letter) outside the brackets. You can spot these questions as they will say **factorise fully** instead of just **factorise**.

Example 2

Factorise fully: $18x+24$

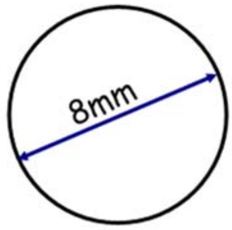
Solution

You could answer $2(9x+12)$ or $3(6x+8)$. However, you would not get full marks as the biggest factor of 18 and 24 is 6. The number outside the bracket must be 6.

Correct answer: $6(3x+4)$

Example 2 – diameter

Calculate the area of this circle.



Solution

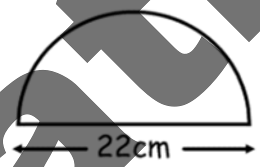
8mm is the diameter, so the radius is 4mm, or *r* = 4.

$$\begin{aligned} A &= \pi r^2 \\ &= \pi \times 4^2 \quad (\text{or } 3\cdot14 \times 4^2) \\ &= 50\cdot26548... \\ &= \underline{50\cdot3\text{mm}^2} \ (1 \text{ d.p.}) \end{aligned}$$

Definition: a semicircle is half of a circle.

Example 3 – semicircle

Calculate the area of this semicircle.



Solution

22cm in this diagram is the *diameter*. This means that the radius is 11cm or *r* = 11cm.

$$\begin{aligned} A &= \pi r^2 \div 2 \\ &= \pi \times 11^2 \div 2 \quad (\text{or } 3\cdot14 \times 11^2 \div 2) \\ &= 190\cdot0663555... \\ &= \underline{190\cdot1\text{cm}^2} \ (1 \text{ d.p.}) \end{aligned}$$

Area of Composite Shapes

Once you know how to find the area of a rectangle, or a triangle, or a circle, you can then work out the area of more complex shapes by splitting them up into rectangles, triangles and circles.

Definition: a **composite shape** is one made by joining two or more other shapes together. In the exam, areas will always be of composite shapes, usually made up of rectangles, squares, triangles or semi-circles (for semi-circles see page 39).

The method to work out the area of a composite shape is always the same:

Step one: split the shape up into smaller, simpler shapes.

Step two: calculate the area of each smaller shape separately.

Step three: either add or take away the areas:

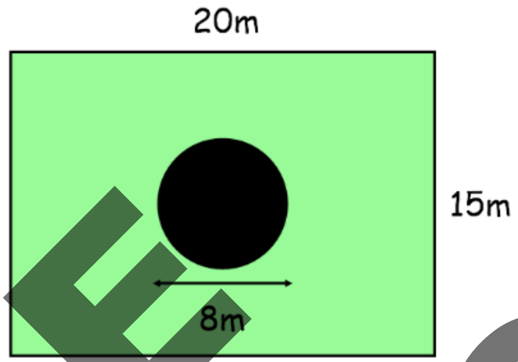
- If the two shapes are joined together, you **add** the areas.
- If one shape is cut out of the other, you **take away** its area.

Example 1

The diagram shows a grass lawn. The lawn is in the shape of a rectangle measuring 15 metres by 20 metres.

There is a circular flowerbed cut out of the middle of the lawn with diameter 8 metres.

Calculate the total area of the grass.



Solution

Area of rectangle:

$$A = LB$$
$$= 20 \times 15$$
$$= 300\text{m}^2$$

Area of circle:

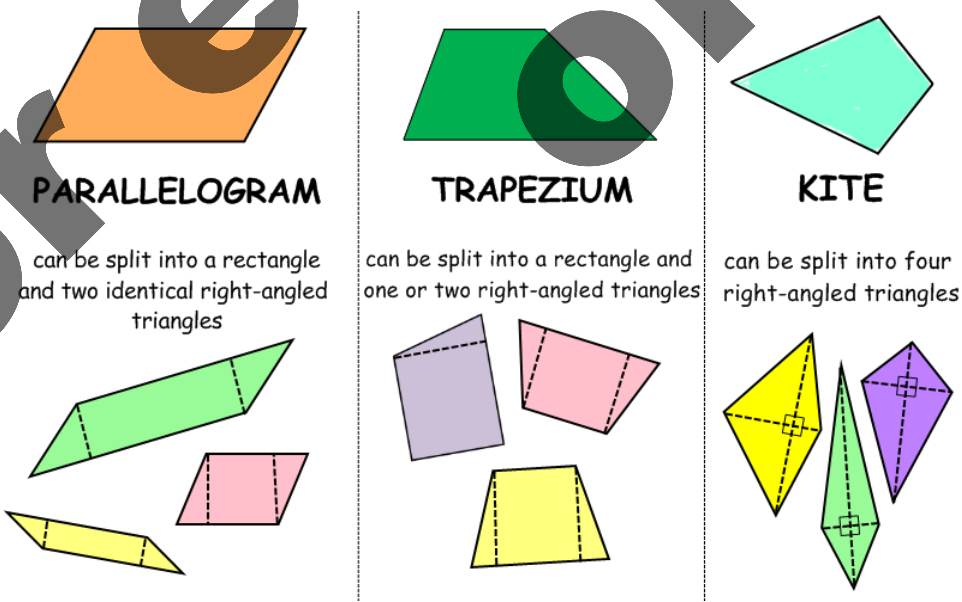
Diameter of circle is 8m so radius is 4m

$$A = \pi r^2$$
$$= \pi \times 4^2$$
$$= 50 \cdot 265 ... \text{m}^2$$

Total area of grass remaining = 300 – 50·265 = <u>249·73m²</u>

You need to be able to work out the area of a **trapezium**, or a **parallelogram** or a **kite** by splitting the shapes up into rectangles and triangles.

Definitions:
- A **quadrilateral** is a shape with four straight sides.
- A **parallelogram** is a quadrilateral with two pairs of parallel sides. It can be split up to be a rectangle and two identical right-angled triangles.
- A **trapezium** is a quadrilateral with one pair of parallel sides. It can be split up to be a rectangle and one or two right-angled triangles.
- A **kite** is a quadrilateral with one line of symmetry. It can be split up to be four triangles (two pairs of equal triangles).



Solution

Diameter is 10cm so radius is 5cm.

$$V = \pi r^2 h$$
$$= \pi \times 5^2 \times 20 \quad (\text{ or } \pi \times 5 \times 5 \times 20 \text{ })$$
$$= 1570 \cdot 796327....$$
$$= \underline{1570 \cdot 8\text{cm}^3} \text{ (1 d.p.)}$$

Surface Areas of 3-d Shapes

A three-dimensional shape has faces, edges and vertices. Given a simple 3-d shape (most likely a cuboid or prism), you are expected to be able to:

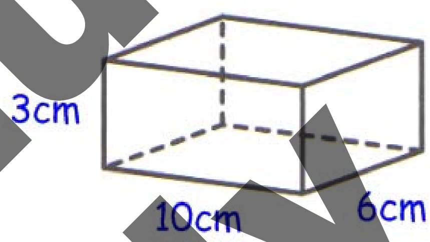
1. state the number of faces, edges and vertices.
2. draw a possible net of the shape.
3. calculate the surface area of the shape (using its net or otherwise) by splitting it up into individual rectangles, triangles and/or circles.

A **cuboid** is made up of six rectangular faces, made up of three pairs of equal faces.

Example 1 – Cuboid

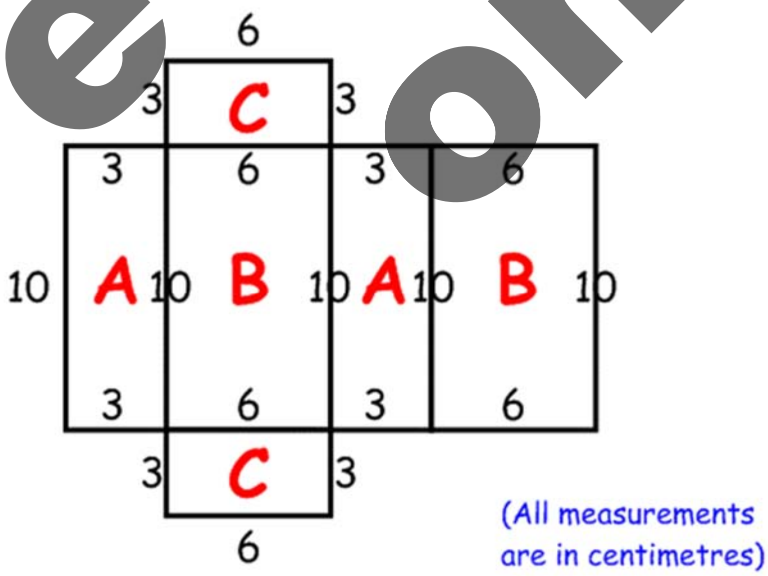
(a) **Draw a possible net for the cuboid shown in the diagram.**

(b) **Calculate the total surface area of the cuboid.**



Solution

(a) A possible net, made up of six rectangles, is drawn below.



Comparing Statistics

The **mean**, **median** and **mode** are averages. They tell us whether a list of numbers is higher or lower on average.

The **range** is NOT an average. Instead it is a measure of spread. It tells us whether a list of numbers is more or less spread out/consistent.

- A lower range means the numbers are more **consistent**.
- A higher range means the numbers are more **varied**.

Example

The temperature in Aberdeen has a mean of 3°C and a range of 5°C.
In London it has a mean of 9°C and a range of 3°C.
Make two comments comparing the temperatures in London and Aberdeen.

Solution

You would get NO MARKS (as you are just stating the obvious) for:

- *"Aberdeen has a lower mean"*
- *"London has a higher mean"*
- *"Aberdeen has a higher range"*
- *"London has a lower range".*

You would get NO MARKS (because your sentence makes no sense) for:

- *"Aberdeen is lower".* (no mention of temperature)
- *"The first one has a lower temperature".* (no mention of Aberdeen or London)
- *"In London it is more consistent".* (no mention of what 'it' is)
- *"The standard deviation in London is more consistent".* (it is the temperature that is more consistent, not the standard deviation).

You WOULD get marks (because you explain what the numbers mean) for:

- *"On average, the temperature in Aberdeen is lower than London and the temperature is less consistent"*
- *"The temperature in London is higher and more consistent than Aberdeen"*

Pie Charts

You need to be able to work out the angles you would use to draw the slices in a pie chart.

To do this:

- Step One: calculate the degrees for 'one slice' using the formula **360 ÷ total**
- Step Two: multiply each frequency by that number.
- Step Three: (optional but highly advisable) check the angles add up to 360°.
- Step Four: draw the pie chart, ensuring your angles are correct to ±2°.
- Step Five: label the slices of your pie chart.

Example 3

Coco's Cake Shop are running a competition. If you buy a cake containing a lucky charm, then you win a prize.

In their Dalkeith shop, they bake 200 cakes and put lucky charms in 6 of them at random. In their Mayfield shop, they bake 160 cakes and put lucky charms in 4 of them.

Sharon is going to buy a cake today. Determine which shop should she buy from to have the best chance of winning a prize. Justify your answer.

Solution

Dalkeith

The probability of finding a lucky charm in a cake is $\frac{6}{200}$ as a fraction.

To convert this to a percentage: $\frac{6}{200} = 6 \div 200 \times 100 = \underline{3\%}$. .

Mayfield

The probability of finding a lucky charm in a cake is $\frac{4}{160}$.

$\frac{4}{160} = 4 \div 160 \times 100 = \underline{2 \cdot 5\%}$.

3% is bigger than 2·5% so the decision is that Sharon should buy a cake from the *Dalkeith* shop for a (slightly) increased chance of winning.

The following *explanations* **should** get a mark. See the examples on page 7 for further guidance on how to write an explanation.

- Dalkeith, because the probability is 3% which is more than 2·5%. *(two numbers and a comparing word).*
- Dalkeith, because 3% > 2·5%. *(two numbers and a comparing symbol).*
- Dalkeith, because the probability is 0·5% higher. *(stating the difference).*

You would **not** get a mark for these explanations:

- Dalkeith. *(no reason given)*
- Dalkeith, because the probability is higher. *(numbers not stated)*
- Dalkeith, because it is 3%. *(only one number stated)*

Relationships Unit

Algebra

Vertical and Horizontal Lines

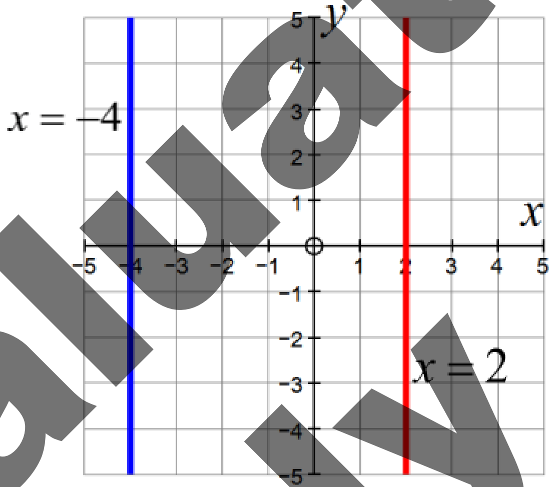
If we have an equation (such as $y = 3x - 5$ or $y = 10 - \frac{1}{2}x$), we can use algebra to plot coordinates of points that match that equation. These points will always lie on a straight line. If we join the points together, we say we have drawn "the graph of the equation".

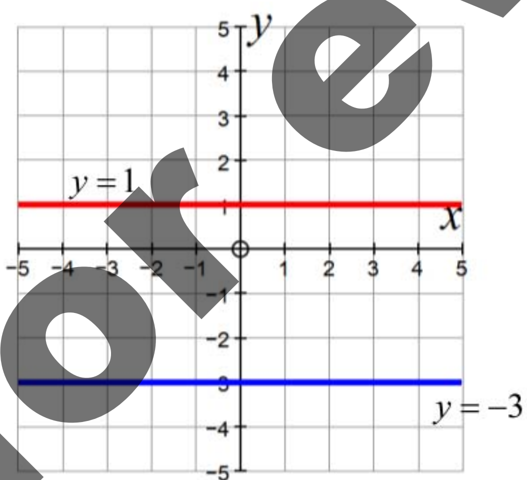
The simplest lines are vertical and horizontal ones. They are very easy to draw. However, because we spend most of the time talking about the more complicated lines, people tend to forget these ones.

Vertical Lines

Vertical lines have equations such as $x = 2$ or $x = -4$ (we say that these equations are "of the form $x = a$" where a can be any number). The diagram on the right shows that:

- the line $x = 2$ is a line going vertically through 2 on the x axis.
- the line $x = -4$ is a line going vertically through –4 on the x axis.





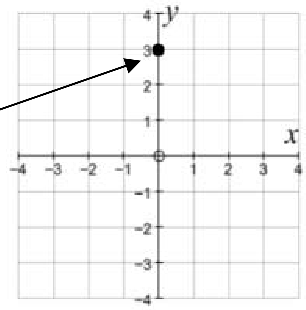
Horizontal lines

Horizontal lines have equations such as $y = 1$ or $y = -3$ (we say that these equations are "of the form $y = b$" where b can be any number). The diagram on the left shows that:

- the line $y = 1$ is a line going horizontally through 1 on the y axis.
- the line $y = -3$ is a line going horizontally through –3 on the y axis.

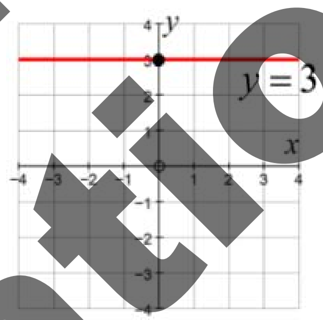
Example 1

Draw the straight line with equation $y=3$



Solution

$y=3$ must go through number 3 on the *y*-axis, so we can mark that point (0, 3) straight away.

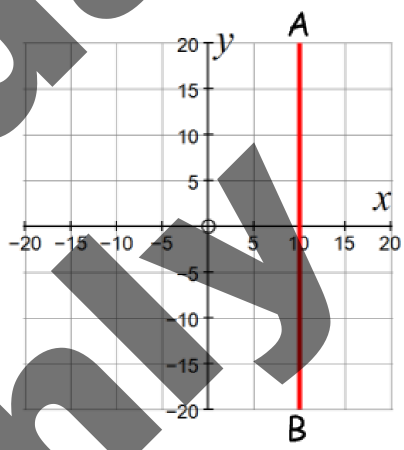


To go through that point, the line must be horizontal, so we can draw the line and complete the graph.

We must remember to give the graph the label $y=3$.

Example 2

The diagram on the right shows the straight line AB. State the equation of AB.



Solution

The line is vertical and goes through 10 on the *x* axis. Therefore, its equation is $x=10$.

Drawing a Straight line from its equation

Other straight line equations that have both *x* and *y* in are more complicated to do. We must use a **table of values** to work out what *y* is for different values of *x*.

When doing a table of values, we can choose any values of *x*. However it makes sense to choose simple numbers, and 0, 1, 2, 3 is a common choice.

Example

Draw the straight line with equation $y=3x-4$.

Solution

Step one: draw up a table of values

x	0	1	2	3
y				

(continued on next page)

- Step two: simplify both sides.
- Step three: divide to solve the resulting equation.
- Final step: double check your answer is correct by substituting it back in to both sides of the original equation and checking both sides give the same answer.

Example 1

Solve algebraically the equation $2a + 5 = 15 - 3a$.

Solution

***Optional first step**: write in "invisible plus signs" in front of anything that does not already have a sign*

Step one: move the '–3*a*' over to the left-hand side where it becomes '+3*a*'. Move '+5' to the right-hand side where it becomes '–5'.

Step two: simplify both sides.

Step three: divide to get the final answer.

$$2a + 5 = 15 - 3a$$
$$+2a + 5 = +15 - 3a$$
$$+2a + 3a = +15 - 5$$
$$5a = 10$$
$$a = \frac{10}{5}$$
$$a = 2$$

Final step: check, by substituting *a* = 2 into both sides of the original equation:

- The left-hand side is 2*a* + 5. If we substitute *a* = 2, we get 2 × 2 + 5, which is 9.
- The left-hand side is 15 – 3*a*. If we substitute *a* = 2, we get 15 – 3 × 2, which equals 9. These are the same, so our answer must be correct.

If the equation contains brackets, we must multiply them out before continuing.

Example 2 – equation with brackets

Solve algebraically the equation $5(y - 1) = 3(y + 3)$.

Solution

***Optional first step:** write in "invisible plus signs" in front of anything that does not already have a sign*

Before you begin: multiply out both brackets.

Step one: move the '+ 3*y*' over to the left-hand side where it becomes '–3*y*'. Move '–5' to the right-hand side where it becomes '+5'.

Step two: simplify both sides

Step three: divide to get the final answer

$$5(y - 1) = 3(y + 3)$$
$$+5y - 5 = +3y + 9$$
$$+5y - 3y = +9 + 5$$
$$2y = 14$$
$$y = \frac{14}{2}$$
$$y = 7$$

Final step: check, by substituting *y* = 7 into both sides of the original equation:

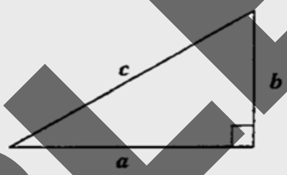
- The left-hand side is $5(y - 1)$. If we replace *y* with 7, we get $5(7 - 1) = 5 \times 6$, which equals 30.
- The left-hand side is $3(y + 3)$. If we replace *y* with 7, we get $3(7 + 3)$, which also equals 30. These are the same, so our answer is correct.

Trigonometry

Pythagoras' Theorem

When you know the length of any two sides of a right-angled triangle you can use Pythagoras' Theorem (usually just known as **Pythagoras**) to calculate the length of the third side without measuring.

This formula <u>is</u> given on the formula sheet for assessments
Theorem of Pythagoras:



$$a^2 + b^2 = c^2$$

Definition: the hypotenuse is the longest side in a right-angled triangle. In the diagram above, the hypotenuse is *c*. The hypotenuse is always opposite the right angle.

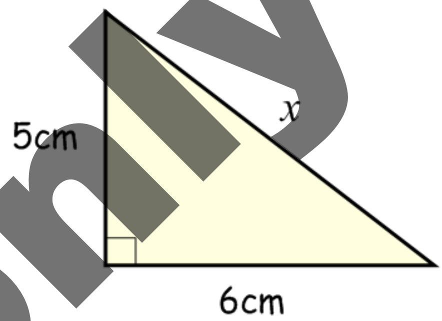
There are three steps to any Pythagoras question:

<u>Step One:</u> square the length of the two given sides.
<u>Step Two:</u> either add or take away:

- To find the length of the longest side (hypotenuse), **add** the squared numbers.
- To find the length of a shorter side, **take away** the squared numbers.

<u>Step Three:</u> square root.

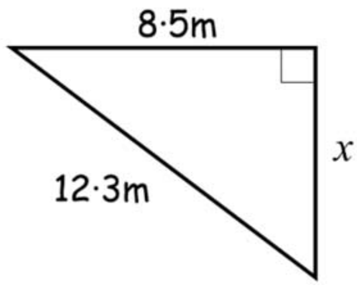
<u>Example 1 – finding the length of the hypotenuse</u>
Calculate the length of *x* in this triangle.
Do not use a scale drawing.



Solution
We are finding the length of *x*.
x is the hypotenuse, so we **add**:

$$x^2 = 5^2 + 6^2$$
$$x^2 = 61$$
$$x = \sqrt{61}$$
$$x = 7 \cdot 81024....$$
$$x = \underline{7 \cdot 81\text{cm}} \text{ (2 d.p.)}$$

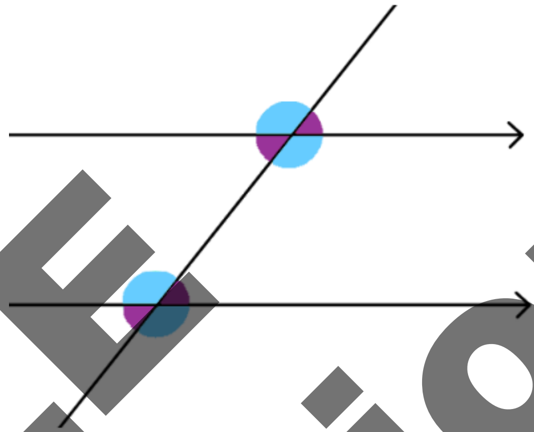
<u>Example 2 – finding the length of a shorter side</u>
Calculate *x*, correct to 1 decimal place.
Do not use a scale drawing.



(Solution is on the next page)

Angles and Parallel Lines

We know two lines are parallel if they both have an arrow on them, such as in the diagram on the right.



When two parallel lines are crossed by a third straight line, the angles at each X shape are identical. In the diagram on the right, the four angles that are shaded darker are all the same size; and the four angles shaded lighter are also all the same size.

Facts

- Two angles on a straight line add up to make 180°.
- Opposite angles in X-shapes are equal.
- Angles in Z shapes (made by parallel lines) are the same.

Angles and Circles

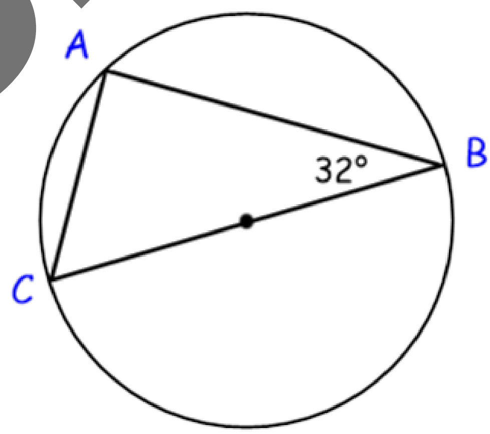
If there is a circle in a diagram containing angles, then there are two extra rules that can help us calculate unknown angles.

One rule relates to a triangle that fills half a circle (i.e. its longest side is a diameter of the circle, and the other side is on the circumference). We refer to the angle opposite the diameter as the **angle in the semicircle**.

Fact
The angle in a semicircle is always a right angle.

Example 1

The diagram shows a circle with diameter BC. Angle ABC is 32°. Calculate the size of angle ACB.



Solution

Angle CAB is an angle in a semicircle, so CAB = 90°.

Then we know ABC is a triangle with angles 90°, 32°. The third angle (ACB) is 180 – 90 – 32 = 58°.

Answer: angle ACB is 58°.

Index of Key Words