

# National 4

## Applications of Mathematics

### Revision Notes



www.dynamicmaths.co.uk

Last updated January 2019

Use this booklet to practise working independently like you will have to in course assessments and the Added Value Unit (AVU).

- Get in the habit of turning to this booklet to refresh your memory.
- If you have forgotten how to do a method, **examples** are given.
- If you have forgotten what a word means, use the **index** (back pages) to look it up.

As you get closer to the final test, you should aim to use this booklet less and less.

#### This booklet is for:

- Students doing the National 4 Applications of Mathematics course.
- Students studying one or more of the National 4 Applications of Maths units: **Numeracy, Geometry and Measures** or **Managing Finance and Statistics**.

#### This booklet contains:

- The most important facts you need to memorise for National 4 Applications of Mathematics.
- Examples that take you through the most common **routine** questions in each topic.
- Definitions of the key words you need to know.

#### Use this booklet:

- To refresh your memory of the method you were taught in class when you are stuck on a homework question or a practice test question.
- To memorise key facts when revising for the Added Value Unit.

The key to revising for a maths test is to do questions, not to read notes. **As well as using this booklet, you should also:**

- Revise by working through exercises on topics you need more practice on – such as revision booklets, textbooks, websites, or exercises suggested by your teacher.
- Work through practice tests.
- Ask your teacher when you come across a question you cannot answer.
- Use resources online (a link that can be scanned with a SmartPhone is on the last page)

**licensed to: SAMPLE COPY (unlicensed)**

© Dynamic Maths ([www.dynamicmaths.co.uk](http://www.dynamicmaths.co.uk)) 2019. All rights reserved.

These notes may be photocopied or saved electronically and distributed to the pupils or staff in the named institution only. They may not be placed on any unsecured website or distributed to other institutions without the author's permission. Queries may be directed to [david@dynamicmaths.co.uk](mailto:david@dynamicmaths.co.uk)

## Contents

<b>Formula Sheet .....</b>	<b>3</b>
<b>Assessment Technique .....</b>	<b>4</b>
'Communication' Marks .....	4
Units .....	5
Rounding .....	5
<b>Numeracy Unit .....</b>	<b>7</b>
<b>Numerical Notation and Units .....</b>	<b>7</b>
<b>Add, Subtract, Multiply, Divide and Rounding .....</b>	<b>8</b>
Written Sums .....	8
Rounding .....	9
Adding and Taking Away Negative Numbers (Integers) .....	10
Multiplying and Dividing Negative Numbers (Integers) .....	11
<b>Fractions and Percentages .....</b>	<b>13</b>
Converting Fractions, Decimals and Percentages .....	13
Calculating Fractions .....	14
Percentages without a calculator .....	14
Percentages with a calculator .....	15
Calculating the Percentage Increase or Decrease .....	15
<b>Length, Area and Volume .....</b>	<b>17</b>
Area of a Rectangle and Triangle .....	17
Volumes of Cubes and Cuboids .....	17
<b>Speed, Distance and Time .....</b>	<b>19</b>
Changing time to a decimal .....	19
Speed, Distance and Time Calculations .....	20
<b>Graphs, Charts and Tables .....</b>	<b>22</b>
Interpreting and Comparing Graphs .....	22
Stem and Leaf Diagrams .....	24
Bar Graphs and Line Graphs .....	25
Probability .....	25
<b>Ratio and Proportion .....</b>	<b>27</b>
Ratio .....	27
Direct Proportion .....	27
<b>Coordinates .....</b>	<b>29</b>
<b>Managing Finance and Statistics .....</b>	<b>30</b>
<b>Finance .....</b>	<b>30</b>
Determining a Financial Position .....	30
Pay .....	31
Choosing the Best Deal .....	34
Currency and Exchange Rates .....	36
Savings and Borrowing .....	37
<b>Statistics .....</b>	<b>39</b>
Frequency Tables .....	39
Range .....	40
Mean, Median and Mode .....	40
Comparing Statistics .....	42
Scatter Graphs .....	43
Pie Charts .....	46
<b>Geometry and Measures .....</b>	<b>47</b>
<b>Measurement .....</b>	<b>47</b>
Tolerance .....	47
Time Management .....	47
Calculating a Quantity Based on a Related Measurement .....	49
Scale Drawing and Navigation .....	49
Container Packing .....	52
<b>Geometry .....</b>	<b>55</b>
Gradient .....	55
Perimeter .....	56
Circumference of a Circle .....	57
Area of a Circle .....	58
Area of Composite Shapes .....	59
Volume of a Prism .....	62
Pythagoras' Theorem .....	63
Enlargement and Reduction by a Scale Factor .....	64
<b>Index of Key Words .....</b>	<b>66</b>

## Formula Sheet

The following formulae are mentioned in these notes and are collected on this page for ease of reference.

**Formulae that are given on the formula sheet in the Added Value Unit** (or in unit assessments)

Topic	Formula(e)	Page Reference
Circumference of a circle	$C = \pi d$	See page 57
Area of a circle	$A = \pi r^2$	See page 58
Gradient	Gradient = $\frac{\text{Vertical height}}{\text{Horizontal distance}}$	See page 55
Pythagoras' Theorem	$a^2 + b^2 = c^2$	See page 63

**Formulae that are not given in assessments**

Topic	Formula(e)	Page Reference
Percentage increase and decrease	$\frac{\text{change}}{\text{original amount}} \times 100$	See page 15
Area of a rectangle	$A = LB$	See page 17
Area of a square	$A = L^2$	See page 17
Area of a triangle	$A = \frac{BH}{2}$ or $A = \frac{1}{2}BH$	See page 17
Volume of a cuboid	$V = LBH$	See page 17
Speed, Distance, Time	$S = \frac{D}{T}$ $T = \frac{D}{S}$ $D = ST$	See page 20
Gross Pay and Net Pay	Net Pay = Gross Pay – Total Deductions	See page 31
Range	Range = Highest – Lowest	See page 40
Mean	Mean = $\frac{\text{Total}}{\text{How many}}$	See page 40
Volume of a prism	$V = Ah$	See page 62

**Example 2 – adding and taking away**

$$\begin{aligned} -6 + 9 &= && \text{start at } -6 \text{ and move } \underline{\text{up}} \text{ } 9. \text{ Answer: } \underline{3} \\ 5 - 7 &= && \text{start at } 5 \text{ and move } \underline{\text{down}} \text{ } 7. \text{ Answer: } \underline{-2} \\ (-2) - 8 &= && \text{start at } -2 \text{ and move } \underline{\text{down}} \text{ } 8. \text{ Answer: } \underline{-10} \end{aligned}$$

Adding a negative number is the same as taking away. When an addition and a subtraction sign are written next to each other, you can “ignore” the addition sign.

**Example 3– adding a negative**

$$\begin{aligned} 2 + (-6) &= && 2 - 6 = \text{start at } 2 \text{ and move } \underline{\text{down}} \text{ } 6. \text{ Answer: } \underline{-4} \\ (-1) + (-7) &= && (-1) - 7 = \text{start at } -1 \text{ and move } \underline{\text{down}} \text{ } 7. \text{ Answer: } \underline{-8} \end{aligned}$$

Taking away a negative number becomes an addition. When two negative signs are written next to each other without a number in between, they become an add sign.

This can be thought of as “**taking away a negative becomes an add**”

**Example 4 – taking away a negative**

$$\begin{aligned} 5 - (-2) &= && 5 + 2 = \underline{7} \\ (-7) - (-2) &= && (-7) + 2 = \text{start at } -7 \text{ and move } \underline{\text{up}} \text{ } 2. \text{ Answer: } \underline{-5} \end{aligned}$$

**Multiplying and Dividing Negative Numbers (Integers)**

Multiplying and dividing integers have *completely different* rules to adding and taking away. To multiply and divide, you do the sum normally (as if there were no negative signs there), and then you decide whether your answer needs to be negative or positive.

When multiplying and dividing:

- If none of the numbers are negative, then the answer is **positive**.
- If one of the numbers is negative, then the answer is **negative**.
- If two of the numbers are negative, then the answer is **positive**.
- If three of the numbers are negative, then the answer is **negative**.

and so on...

In short, the rules are:

+ multiplied by + gives you +  
 – multiplied by + gives you –  
 + multiplied by – gives you –  
 – multiplied by – gives you +

+ divided by + gives you +  
 – divided by + gives you –  
 + divided by – gives you –  
 – divided by – gives you +

**Example 1 – multiplication**

$(-5) \times 4 =$	$-20$	$\rightarrow$	<i>If <b>one</b> of the numbers is negative, then the answer is <b>negative</b>.</i>
$60 \times (-2) =$	$-120$	$\rightarrow$	
$(-3) \times (-10) =$	$+30$ (or just 30)	$\rightarrow$	<i>If <b>two</b> of the numbers are negative, then the answer is <b>positive</b>.</i>
$(-2) \times 3 \times (-4) =$	$24$	$\rightarrow$	
$(-2) \times (-3) \times (-4) =$	$-24$	$\rightarrow$	<i>If <b>three</b> of the numbers are negative, then the answer is <b>negative</b>.</i>

In particular, if you square a negative number, **the answer always has to be positive**, because you are multiplying two negative numbers.

---

#### Example 2 – squaring

$$(-6)^2 = (-6) \times (-6) = 36$$

$$(-10)^2 = (-10) \times (-10) = 100$$

→ If **two** of the numbers are negative, then the answer is **positive**.

---

#### Example 3 – dividing

$$(-28) \div 4 = -7$$

$$50 \div (-5) = -10$$

$$(-80) \div (-10) = +8 \text{ (or just 8)}$$

→ If **one** of the numbers is negative, then the answer is **negative**.

→ If **two** of the numbers are negative, then the answer is **positive**.

Examples 1 and 2**Calculate 75% of 480cm.****Calculate  $33\frac{1}{3}\%$  of £330.****Solution**

75% of 480cm

$$= \frac{3}{4} \text{ of } 480\text{cm}$$

$$= 480 \div 4 \times 3 = \underline{360\text{cm}}$$

 $33\frac{1}{3}\%$  of £330

$$= \frac{1}{3} \text{ of } £330$$

$$= £330 \div 3 \times 1 = \underline{£110}$$

Other percentages can be worked out without a calculator by finding 1% or 10% first

For example to find 30%: find 10% first, then multiply the answer by 3.

To find 4%: find 1% first then multiply the answer by 4.

Examples 3 and 4**Calculate 40% of £120.****Calculate 7% of 3000kg.****Solution**

$$10\% \text{ of } £120 = £12$$

$$\text{so } 40\% \text{ of } £120 = 12 \times 4 = \underline{£48}$$

$$1\% \text{ of } 3000\text{kg} = 30\text{kg}$$

$$\text{so } 7\% \text{ of } 3000\text{kg} = 30 \times 7 = \underline{210\text{kg}}$$

**Percentages with a Calculator**

For every question, there are two ways of doing it. Use the one you are happiest with.

Question	Method 1 Divide and Multiply	Method 2 Decimal	Answer
27% of £360	$360 \div 100 \times 27$	$0.27 \times 360$	<b>£97.20</b>
3% of £250	$250 \div 100 \times 3$	$0.03 \times 250$	<b>£7.50</b>
17.5% of £4200	$4200 \div 100 \times 17.5$	$0.175 \times 4200$	<b>£735</b>
4.2% of £360	$360 \div 100 \times 4.2$	$0.042 \times 360$	<b>£15.12</b>

Example

A car is normally priced at £8800. In a sale, the price has been reduced by 12%. Calculate the new price of the car.

**Solution**

$$12\% \text{ of } £8800 = 0.12 \times 8800 \text{ [or } 8800 \div 100 \times 12] = £1056$$

$$\text{New price} = 8800 - 1056 = \underline{£7744.}$$

**Calculating the Percentage Increase or Decrease**

To find the percentage increase or decrease, we use the method for changing fractions to percentages outlined on page 14.

In these questions, we work out the percentage of the **original** amount. The steps are:

## Graphs, Charts and Tables

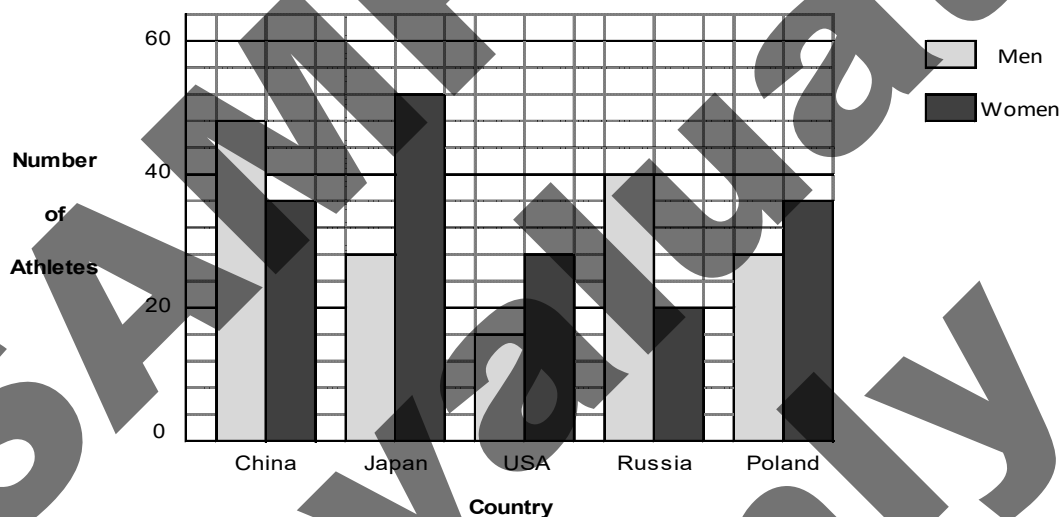
### Interpreting and Comparing Graphs

To pass the National 4 numeracy unit, you need to be able to obtain information from at least two different types of diagram. These could include any sort of graph, chart or table – including a frequency table, a table of information, a bar chart, line graph, pie chart (see page 23), stem and leaf diagram (see page 24) or scatter graph (see page 43).

For the National 4 Applications of Mathematics Added Value Unit you may be required to compare information and calculate differences from a graph or graphs. This is most likely to involve either a **bar graph** or a **pie chart**, though it could involve other graphs too.

#### Example 1 – Bar Chart, sample Added Value Unit question

**The graph shows the number of athletes from five countries taking part in an international Sports Tournament.**



- (a)  $\frac{2}{3}$  of the male athletes from China were swimmers. Calculate the number of swimmers.
- (b) How many more athletes were from Japan compared to the USA?

#### Solution

- (a) The graph tells us that there were 48 male athletes from China in total.

To find  $\frac{2}{3}$  we divide by 3 and multiply by 2:

$$48 \div 3 \times 2 = \underline{32} \text{ athletes.}$$

- (b) The graph shows that there were 28 men and 52 women from Japan. This is a total of 80 athletes.  
The graph also shows that there were 16 men and 28 women from the USA. This is a total of 44 athletes.  
 $80 - 44 = 36$ , so there were 36 more athletes from Japan compared to the USA.

## Overtime

### Definitions:

- A worker's **basic hours** are the hours that they *must* work each week (or each month etc.). *e.g. John works a basic 35 hour week.*
- Overtime** hours are any extra hours that a worker works in addition to their basic hours. *e.g. Jackie works a basic 28 hour week. If Jackie works 31 hours in a week, then she has done 3 hours of overtime.*

You get paid more for each hour of overtime you work than you do for your basic hours. There are two common ways of doing this:

- Double time – where the hourly wage is doubled for overtime hours.
- Time-and-a-half – where you get half as much again for overtime hours.

To work out overtime, the calculation is:

- $\dots \times 2$  (for **double time**)
- $\dots \times 1.5$  (for **time-and-a-half**)

### Example 3 – overtime

**Janet works part time in a chemist and works a basic 14-hour week. Janet is paid a basic rate of £5.30 per hour, and gets time-and-a-half for overtime. Calculate Janet's gross pay in a week where she works 17 hours.**

### Solution

Janet works 14 basic hours, and 3 overtime hours

Basic hours:  $14 \times £5.30 = £74.20$

Overtime:  $3 \times £5.30 \times 1.5 = £23.85$

Gross pay:  $£74.20 + £23.85 = \underline{£98.05}$

### Example 4 – completing a pay slip given a mixture of information

**Jen works in a newsagent. She gets paid a basic wage of £6.30 an hour. When she works overtime, she gets double time.**

**In a particular week:**

- Jen works 15 hours for basic pay.
- Jen works 7 hours overtime.
- Jen pays 20% of her gross pay as tax.
- Jen pays 6% of her gross pay as National Insurance (NI).

**Complete the payslip shown to calculate Jen's net pay for that week.**

Payslip			
Name	Employee No.	Week	NI Number
Jen	0034	50	HT867473A
Basic Pay	Overtime Pay		Gross Pay
		-	
Tax	National Insurance	Pension	Total Deductions
		£0.00	
			Net Pay



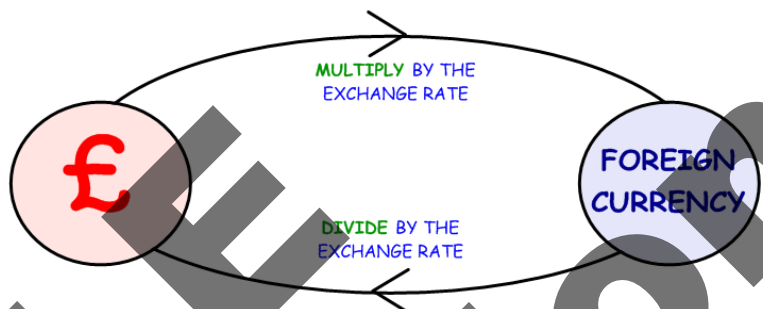
## Currency and Exchange Rates

To convert money from one currency to another, we need an exchange rate.

In all questions at National 4 level, the exchange rate will be expressed in terms of pounds (e.g. £1 = \_\_\_\_).

In these notes, we will use the term 'foreign money' to refer to any currency other than pounds.

Exchange rate: £1 = \_\_\_\_



### Example 1 – changing from pounds into foreign currency

Janet changes £250 into Euros. The exchange rate is £1 = €1.13. Calculate how many Euros Janet will get.

#### Solution

To change from pounds into foreign money, we **multiply** by the exchange rate:

$$250 \times 1.13 = 282.5 = \underline{\pounds 282.50} \text{ (units and two decimal places are essential)}$$

### Example 2 – changing back into pounds

Harry went to the USA with \$1500. Whilst in the USA, he spent \$700.

When returning from the USA, Harry changes his money back into pounds. The exchange rate is £1 = \$1.27. Calculate how many pounds Harry will receive.

#### Solution

Harry is left with \$1500 – \$700 = \$800. We need to change \$800 to pounds.

To change from foreign money back into pounds, we **divide** by the exchange rate:

$$800 \div 1.27 = 629.921... = \underline{\pounds 629.92} \text{ (units and two decimal places are essential)}$$

Some companies charge **commission** when they convert money. This is usually a percentage of the money which they keep as their payment.

### Example 3 – with commission

Maisie is changing £800 into Japanese Yen at the bank. The exchange rate is £1 = 138 Yen (¥). The bank charge 2% commission. Calculate how many Yen Maisie receives.

#### Solution

To change money from pounds into foreign money, we **multiply** by the exchange rate:

$$800 \times 138 = 110\,400 \text{ Yen}$$

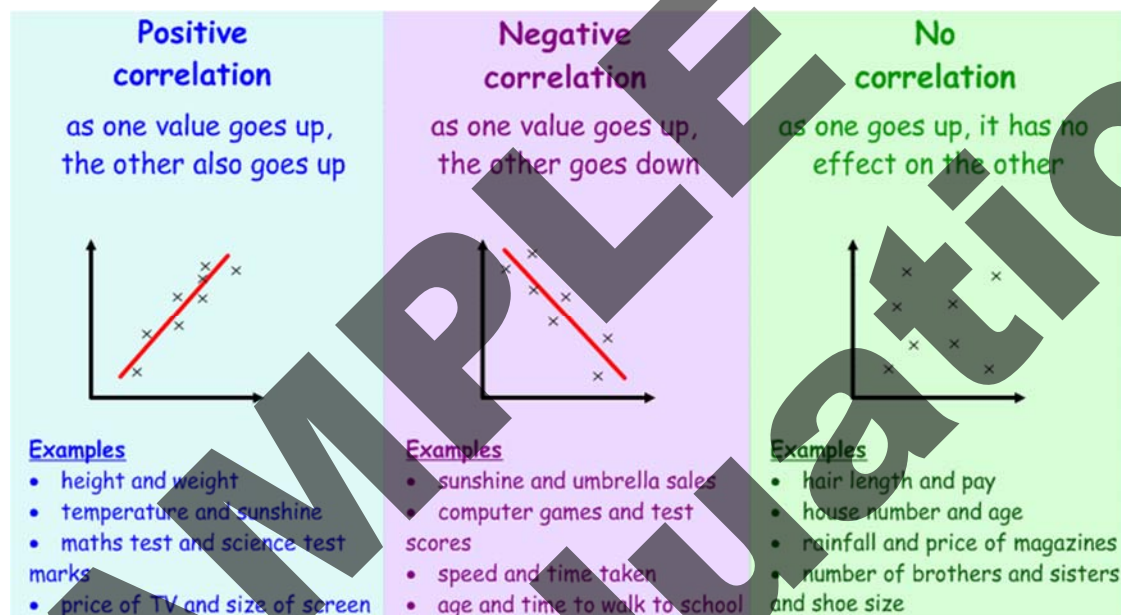
$$2\% \text{ of } 110\,400 = 0.02 \times 110\,400 = 2208, \text{ so the bank's commission is } 2208 \text{ Yen.}$$

$$\text{Maisie receives } 110\,400 - 2208 = \underline{108192 \text{ Yen.}}$$

## Scatter Graphs

A scatter graph displays two sets of linked data on one diagram.

**Definition:** the **correlation** between two data set refers to the relationship (if any) between the numbers. A scatter graph is good for showing correlation. Correlation can be **positive** (going up), **negative** (going down), or **none**.



**Definition:** a **line of best fit** is line drawn on to a scatter graph that shows the correlation of the graph. The straight lines drawn above for positive and negative correlation are examples of lines of best fit.

The line of best fit should go:

- go through the middle of the points, with roughly the same number of points above and below the line
- in the same direction that the points are laid out on the page. **Do not "join the dots"**.
- The line of best fit does not have to go through the origin.

**Tip:** Try and make sure there are roughly the same number of points 'above' and 'below' the line. If there are significantly more points on one side of the line, you won't be able to get the mark.

You will *always* be asked to draw the line of best fit in a scatter graph question in a maths exam. Once you have drawn the line, you will always be asked to *use it*.

### Example 1

A gift shop records the temperature each day for 13 days. They also record how many scarves they sell each day. The results are shown in the table.  
Construct a scatter graph to display this information.

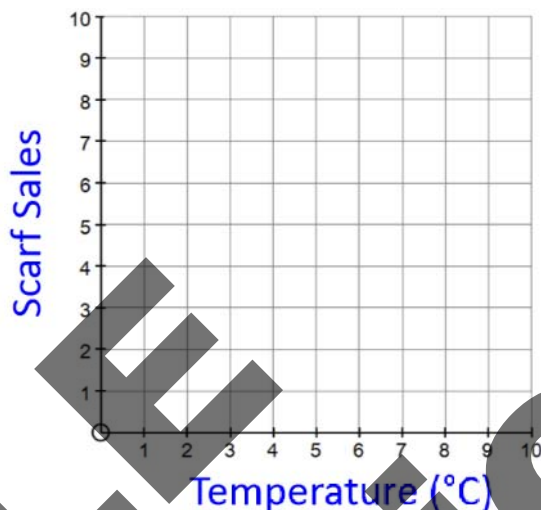
Temperature (°C)	5	4	3	5	7	5	7	1	1	4	2	0
Scarf Sales	3	6	5	4	3	2	0	8	7	4	6	9

**Solution**Step one – draw your axes.

It is important to:

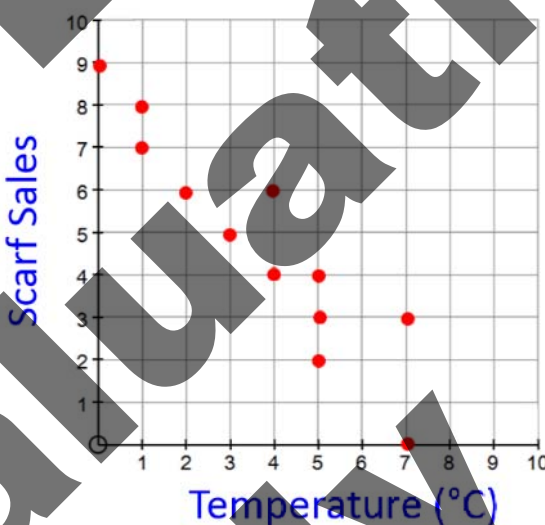
- Label the axes.
- Ensure the numbers go up in equal amounts.

One possible set of axes is shown on the right.

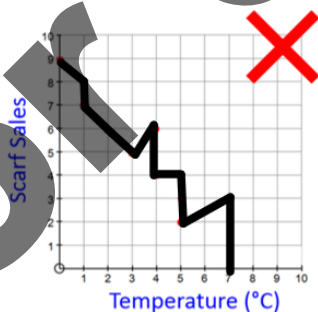
Step two – plot the points.

A possible finished graph is shown on the right.

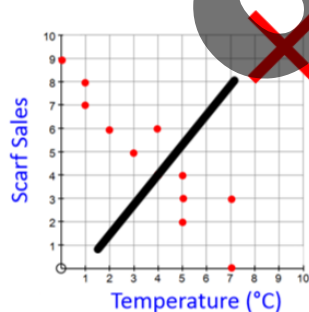
From this graph, we can see there is **negative correlation** between temperature and scarf sales.

Example 2

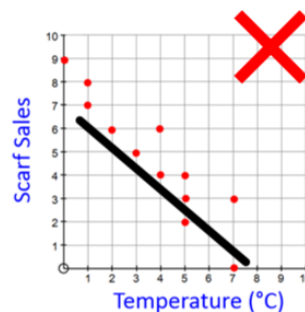
Draw a line of best fit on the scatter graph from example 1.

**Solution**The three lines of best fit below would be marked wrong:

Joining the dots:  
WRONG



Not in same direction as  
the points: WRONG

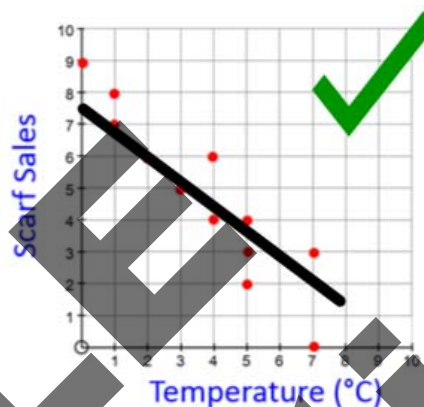
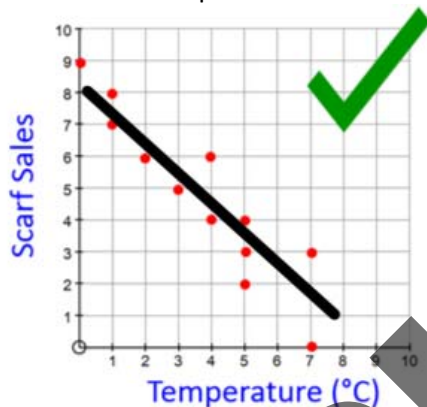


Not through the middle  
of points (too low):  
WRONG

*(continued on next page...)*

(Example 2 continued)

Any of the two lines of best fit below should be marked **correct** as they go roughly through the middle of the points, and roughly in the same direction as the points. There are other possible correct lines as well.



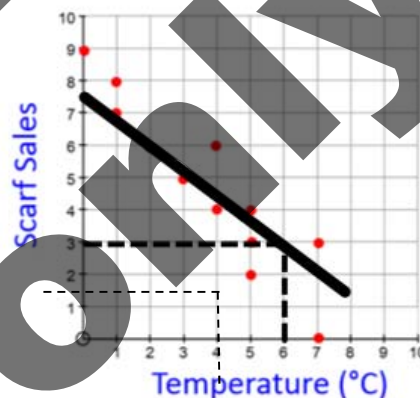
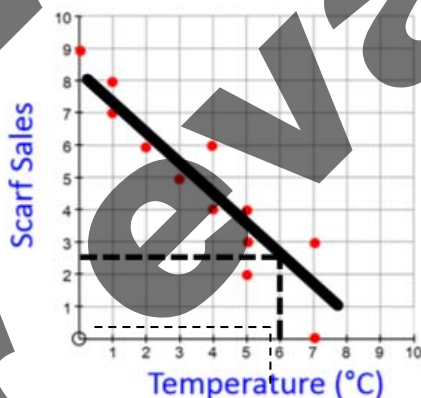
### Example 3

On the next day, the temperature is  $6^{\circ}\text{C}$ . Using your line of best fit, estimate how many scarves the shop will sell.

### Solution

The key words here are *using your line of best fit*. If your answer matches with your line, you get the mark. If it doesn't match with your line, you don't get any marks: Simple as that.

The correct answer will depend on your graph. You need to draw lines on your graph at  $6^{\circ}\text{C}$ , and to see where they meet the line of best fit. For the two examples above, this would look like this:



If your line of best fit was the one on the left, your answer would be 2.5, which you could then round to either 2 or 3 scarves. If your line of best fit was the one on the right, your answer would be 3 scarves.

It does not matter that these answers are different: remember the question only asked for an *estimate*. The important thing is that it matches your line of best fit.

**Solution**

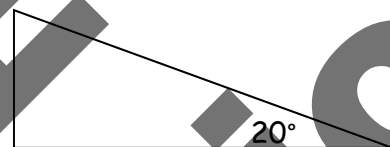
The scale is 1cm = 20m. The scale factor in this question is **20**.

The real-life length of the horizontal line is 100m.

To calculate the distance on the page, we divide by the scale factor and change the units of the answer to centimetres:

$$100 \div 20 = \underline{5\text{cm}}. \text{ So we draw a horizontal line 5cm long.}$$

Now we draw a  $20^\circ$  angle at the right-hand end of the horizontal line using a protractor, and a  $90^\circ$  angle at the left-hand end. (note – the actual diagram here will not be to scale)



Now to work out the real-life height of the oil well, we measure the vertical line in our scale drawing. If our diagram is correct, we should find it is 1.8cm tall.

To calculate the real-life height, we multiply by the scale factor and change the units to metres:

$$1.8 \times 20 = 36, \text{ so the real-life height is } \underline{36\text{m}}.$$

For unit assessments, you need to be able to plot a **navigation** course showing a journey when given distances and three-figure bearings. A three-figure bearing is a way to describe a direction more accurately than a compass.

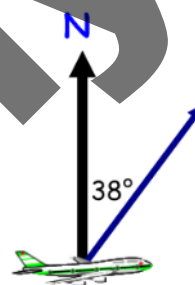
Always start from North and move in a clockwise direction. You must also use three figures, so we write  $085^\circ$  instead of  $85^\circ$ , and we write  $002^\circ$  instead of  $2^\circ$ .

**Example 2 – bearings**

**What bearing is the plane in the diagram flying on?**

**Solution**

The plane is flying on a bearing  $038^\circ$ .



When drawing your own navigation course you need to use the scale to work out how long the lines in your diagram must be.

**Example 3 – drawing a navigation course**

**Some soldiers are marching across the countryside. From the start, they march:**

- 2400m on a bearing of  $040^\circ$  to reach a lake;
- 800m on a bearing of  $200^\circ$  to reach a hut.

**Using the scale 1cm = 200m, construct a scale drawing of the route.**

**Solution**

The scale is 1cm = 200m. The scale factor in this question is **200**.

*(continued on next page)*

(Example 3 continued)

**Task One:** calculate the lengths needed for the drawing (the angles do not change).

To calculate the distance on the page, we divide by the scale factor and change the units of the answer to centimetres:

1<sup>st</sup> leg of journey:

$$2400 \div 200 = 12,$$

so we will draw a line **12cm** long on a bearing of **040°** (40° clockwise from North).

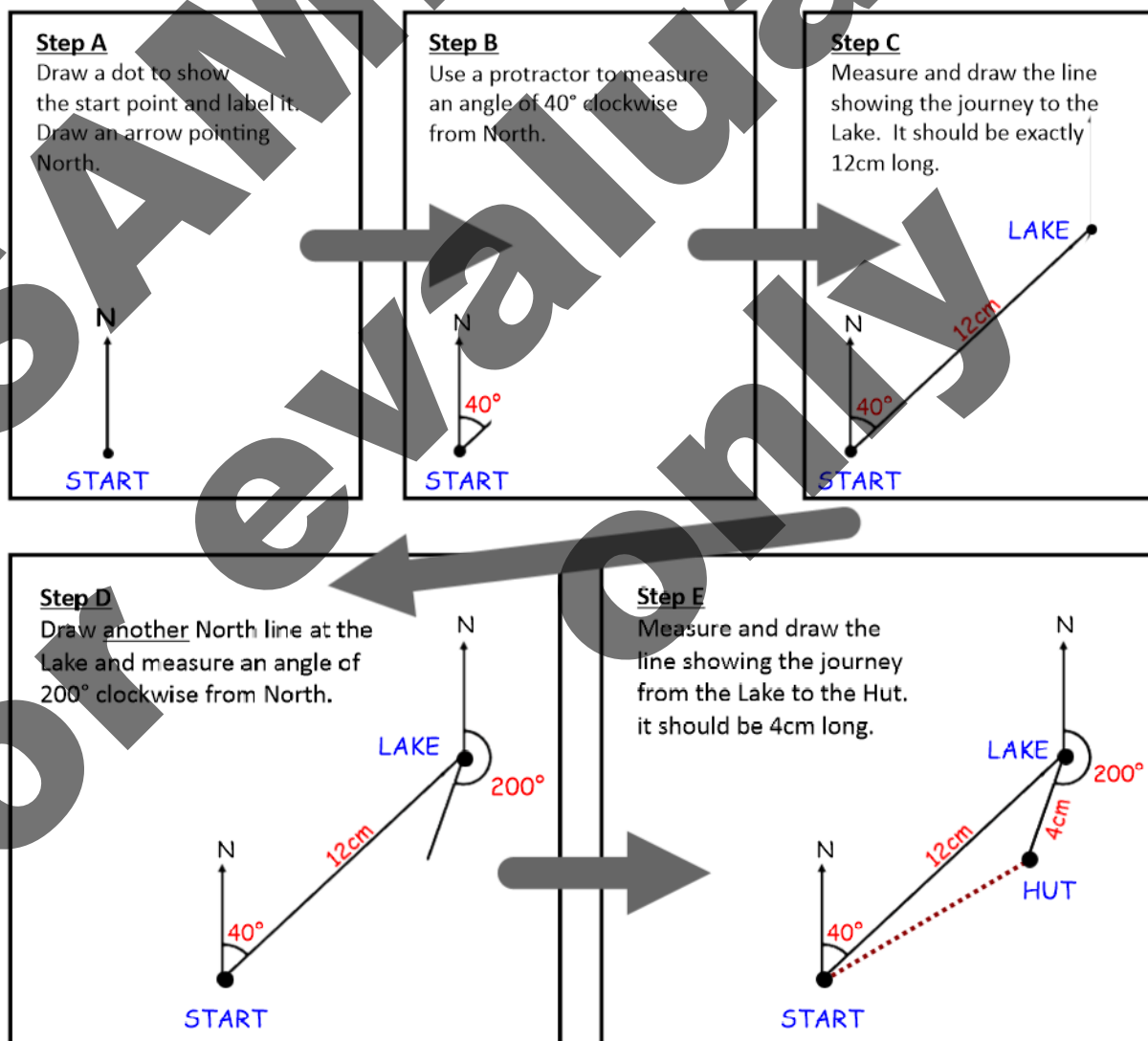
2<sup>nd</sup> leg of journey:

$$800 \div 200 = 4,$$

so we will draw a line **4cm** long on a bearing of **200°** (200° clockwise from North).

**Task Two:** draw your diagram accurately.

The steps involved in drawing the route are outlined here. The finished diagram should look something like the **Step E** picture. Always annotate (label) your diagram thoroughly with all lengths, angles and place names.





## Container Packing

You need to be able to work out how to pack smaller three-dimensional objects inside larger containers. When doing so, we must bear several factors in mind:

- It is *essential* that none of the edges of the smaller objects end up being too big for the larger container.
- It is OK to have extra space left over. However, we want as little unused space as possible as unused space could result in wasted money to a business.
- Some objects may have to be stacked a certain way up so that they do not break.

To find out how many objects fit in, we need to do a division sum with the lengths of the objects and the length of the container. It is not possible to have a fraction of an object so if the answer is a decimal/fraction we must round **down** (never up) to the nearest whole number.

### Example 1

A tin of beans has diameter 8.5cm.

A supermarket shelf measures 120cm. Calculate the largest number of tins that can be fitted in one row on the shelf?



8.5cm

### Solution

$120 \div 8.5 = 14.111$ , so 14 tins can be fitted in a row.

There are two types of question you are likely to be asked:

- Type 1: **how many** objects can you fit in? For example you might be asked how many DVDs of a given size could fit on a bookshelf; or how many containers could fit inside a lorry.
- Type 2: **find a way** to arrange differently sized items. For example you might be shown the sizes of a number of different sized packages boxes and asked to find a way of arranging them inside another, larger, box so that they all fit.

### Example 2 – how many

Books are 3cm wide and 22cm tall. The books need to be stacked into a bookcase.

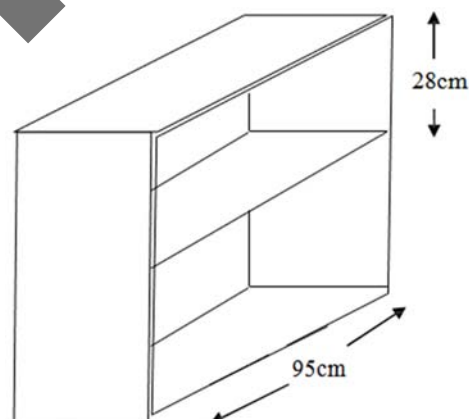
The diagram on the right shows the size of the bookcase. It has two shelves.



- (a) If books are stacked vertically as shown in the picture on the left, calculate how many books will fit on the shelf.



- (b) If books are stacked flat as shown in the picture on the left, calculate how many books will fit on the shelf.



## Circumference of a Circle

### Definitions:

- the **diameter** of a circle is the length all the way across a circle, passing through the centre.
- the **radius** is half of the diameter.
- the **circumference** is the curved length around the outside of a circle. It is a special name for the perimeter of a circle.

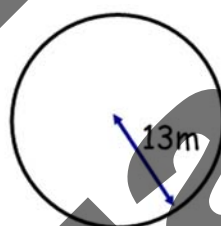
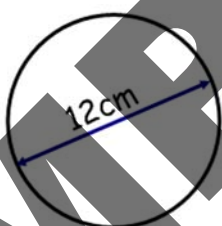
This formula is given on the formula sheet for unit assessments

**Circumference of a circle:**

$$C = \pi d$$

### Example 2 – circumference of a circle

Calculate the circumferences of these two circles:



### Solutions

The *diameter* is 12cm, so  $d = 12$

$$C = \pi d$$

$$= \pi \times 12 \quad (\text{or } 3.14 \times 12)$$

$$= 37.69911184...$$

$$= \underline{37.7\text{cm}} \quad (1 \text{ d.p.})$$

In this circle, the *radius* is 13m

so the *diameter* is 26m, i.e.  $d = 26$

$$C = \pi d$$

$$= \pi \times 26 \quad (\text{or } 3.14 \times 26)$$

$$= 81.68140899...$$

$$= \underline{81.68\text{m}} \quad (2 \text{ d.p.})$$

You may come across more difficult examples that involve quarter and half circles, the next example ask you to calculate the **perimeter** of the shapes. The word 'circumference' refers only to the curved length. Therefore, to work out the perimeter, you also need to add on any straight lengths.

### Example 3 – perimeter of a circle

Calculate the **perimeter** of the quarter-circle shown.

### Solution

7cm is the radius, so the diameter is 14cm, i.e.  $d = 14$ .

The shape is a quarter circle, so we divide the circumference by 4.

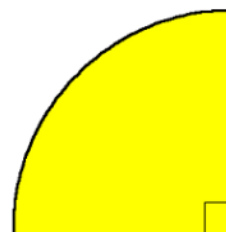
**Step one:** calculate the circumference:

$$C = \pi d \div 4$$

$$= \pi \times 14 \div 4 \quad (\text{or } 3.14 \times 14 \div 4)$$

$$= 10.9955...$$

$$= \underline{11.0\text{cm}} \quad (1 \text{ d.p.})$$



7cm

(continued on the next page...)



*(Example 3 continued)***Step two:** calculate the perimeter by adding on the straight lengths

$$\begin{aligned} \text{Perimeter} &= \text{arc} + \text{---} + \text{---} \\ \text{Perimeter} &= 11.0 + 7 + 7 = \underline{25.0\text{cm}} \end{aligned}$$

## Area of a Circle

**Definition:** the **area** of a 2d shape is a measure of the amount of space inside it.

This formula is given on the formula sheet for unit assessments

**Area of a circle:**

$$A = \pi r^2$$

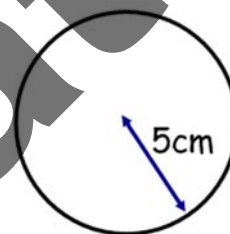
Example 1 – radius

**Calculate the area of this circle.**

**Solution**

The radius of this circle is 5cm, so  $r = 5$ .

$$\begin{aligned} A &= \pi r^2 \\ &= \pi \times 5^2 \quad (\text{or } 3.14 \times 5^2) \\ &= 78.53981634... \\ &= \underline{78.5\text{cm}^2} \quad (1 \text{ d.p.}) \end{aligned}$$



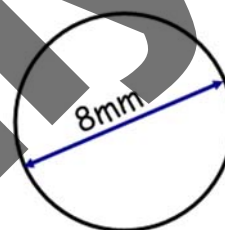
Example 2 – diameter

**Calculate the area of this circle.**

**Solution**

8mm is the diameter, so the radius is 4mm, or  $r = 4$ .

$$\begin{aligned} A &= \pi r^2 \\ &= \pi \times 4^2 \quad (\text{or } 3.14 \times 4^2) \\ &= 50.26548... \\ &= \underline{50.3\text{mm}^2} \quad (1 \text{ d.p.}) \end{aligned}$$



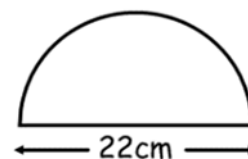
**Definition:** a semicircle is half of a circle.

Example 3 – semicircle

**Calculate the area of this semicircle.**

**Solution**

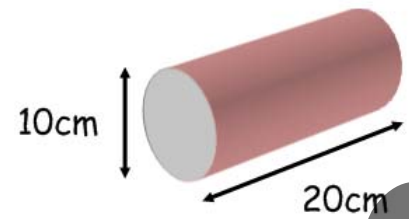
22cm in this diagram is the *diameter*. This means that the radius is 11cm or  $r = 11\text{cm}$ .



*(Continued on next page...)*

Example 2 – cylinder**Calculate the volume of the cylinder shown.****Solution**

The height of this prism is the distance from one (circular) end to the other. In this cylinder, the height is 20cm.

Step 1: Work out the area of the cross-section

In this shape, the cross-section is a circle. The formula for the area of a circle is  $A = \pi r^2$ .

**Important:** you will use a different formula in each question, depending on whether the cross section is a rectangle, square, triangle, circle, semicircle etc.

Diameter is 10cm so radius is 5.

$$\begin{aligned} A &= \pi r^2 \\ &= \pi \times 5^2 \\ &= 78.539... \text{cm}^2 \end{aligned}$$

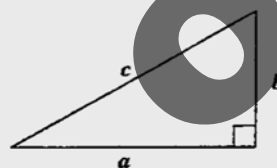
Step 2: Use the formula for volume of a prism:

$$\begin{aligned} V &= Ah \\ &= 78.539 \times 20 \\ &= 1570.796 \\ &= \underline{1570.8 \text{cm}^3} \end{aligned}$$

**Pythagoras' Theorem**

When you know the length of any two sides of a right-angled triangle you can use Pythagoras' Theorem (usually just known as **Pythagoras**) to calculate the length of the third side without measuring.

This formula is given on the formula sheet for assessments

**Theorem of Pythagoras:**

$$a^2 + b^2 = c^2$$

**Definition:** the hypotenuse is the longest side in a right-angled triangle. In the diagram above, the hypotenuse is c. The hypotenuse is always opposite the right angle.

There are three steps to any Pythagoras question:

Step One: square the length of the two given sides.

Step Two: either add or take away:

- To find the length of the longest side (hypotenuse), **add** the squared numbers.
- To find the length of a shorter side, **take away** the squared numbers.

Step Three: square root.

**Example 1 – finding the length of the hypotenuse**

**Calculate the length of  $x$  in this triangle.**  
**Do not use a scale drawing.**

**Solution**

We are finding the length of  $x$ .  
 $x$  is the hypotenuse, so we **add**:

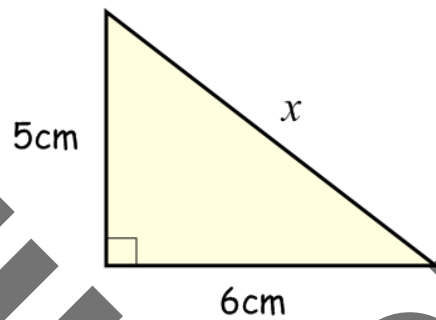
$$x^2 = 5^2 + 6^2$$

$$x^2 = 61$$

$$x = \sqrt{61}$$

$$x = 7.81024....$$

$$x = \underline{7.81\text{cm}} \text{ (2 d.p.)}$$

**Example 2 – finding the length of a shorter side**

**Calculate  $x$ , correct to 1 decimal place.**  
**Do not use a scale drawing.**

**Solution**

We are finding the length of  $x$ .  
 $x$  is a smaller side, so we **take away**:

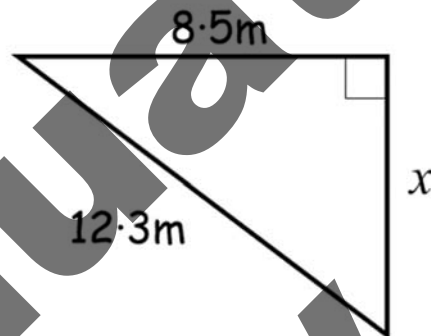
$$x^2 = 12.3^2 - 8.5^2$$

$$x^2 = 79.04$$

$$x = \sqrt{79.04}$$

$$x = 8.8904....$$

$$x = \underline{8.9\text{cm}} \text{ (1 d.p.)}$$

**Enlargement and Reduction by a Scale Factor****Example 1**

The diagram shows a cuboid

- Calculate the volume of the cuboid.
- Each side of the cuboid is enlarged by a scale factor of 3.  
Calculate the new volume of the enlarged cuboid.

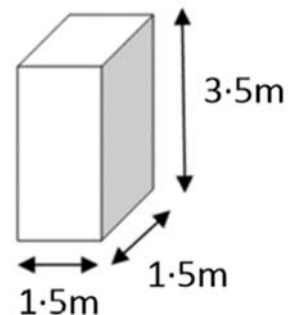
**Solution**

- We find volume using  $V = LBH$

$$V = LBH$$

$$= 1.5 \times 1.5 \times 3.5$$

$$= 7.875\text{m}^3$$



## Index of Key Words

Add and Subtract		Frequency Table .....	39
Written .....	8	Gradient .....	55
Area .....	58	Grams .....	7
of a circle .....	58	Graphs and Charts	
of a composite shape .....	59	Bar Graphs .....	22, 25
of a rectangle .....	17	Line Graphs .....	22, 25
of a semicircle .....	58	Pie Charts .....	23, 46
of a triangle .....	17	Scatter Graph .....	43
Quadrilateral .....	59	Stem and Leaf Diagram .....	24
Average Gradient .....	See Gradient	Trend .....	25
Bar Graphs .....	22, 25	Gross Pay .....	31
Basic hours .....	33	Grouped Frequency Table .....	39
Best Deal .....	34	Hire Purchase (HP) .....	38
Best Fit Line .....	44	Hypotenuse .....	63
Celsius .....	7	Income .....	30, 31
Centimetres .....	7	Instalments .....	38
Circle		Integers .....	10
Area .....	58	multiplying and dividing .....	11
Circumference .....	57	squaring .....	12
Perimeter .....	57	taking away a negative number .....	11
Circumference .....	57	Interest Rate .....	37, 38
Commission .....	32, 36	Kilograms .....	7
Comparing		Kilometres .....	7
Probabilities .....	26	Kite .....	59
Comparing Statistics .....	42	Length .....	7
Composite shape .....	59	Line Graphs .....	22, 25
Container Packing .....	52	Line of Best Fit .....	44
Coordinates .....	29	Litres .....	7, See Volume
Correlation .....	43	Loans .....	37
Cross-section .....	62	Loss .....	30
Cuboid .....	17	Maximum (tolerance) .....	47
Cylinder .....	62	Mean .....	40
Decimals		Measurement	
change to fraction .....	13	Tolerance .....	47
convert to percentage .....	13	Median .....	41
Deductions .....	31	Metres .....	7
Deposit .....	38	Millilitres .....	7, See Volume
Direct Proportion .....	27	Millimetres .....	7
Divide		Minimum (tolerance) .....	47
Written .....	8	Mode .....	41
Double time (overtime) .....	33	Multiply	
Enlargement .....	64	Written .....	8
Expenditure .....	30	National Insurance .....	31
Fahrenheit .....	7	Negative numbers .....	10
Financial Statement .....	30	multiplying and dividing .....	11
First-fit algorithm .....	53	squaring .....	12
Formula (using) .....	49	taking away a negative number .....	11
Fractions		Net Pay .....	31
change to percentage .....	14	NI See National Insurance	
convert to decimal .....	13	Origin .....	29

Overtime.....	33	Scale Factor.....	49, 64
p.a.....	See per annum	Scales	
Parallelogram .....	59	Maps and Diagrams.....	49
Pay .....	31	Scatter Graphs .....	43
per annum .....	37	Estimating a value .....	45
Percentages .....	14	Line of Best Fit.....	44
change to fraction.....	13	Semicircle.....	58
convert to decimal.....	13	Significant figure .....	10
Increase and decrease .....	15	Square	
what is the percentage? .....	14	Area.....	17
with a calculator .....	15	Stem and Leaf Diagram.....	24
without a calculator.....	14	Storage (Container Packing) .....	52
Perimeter.....	56	Temperature.....	7
Circle .....	57	Term (of loan) .....	38
Pie Charts.....	23, 46	Time .....	7, 47
Prism.....	62	Time and a half .....	33
Probability .....	25	Time Intervals .....	19
Profit.....	30	Time Management .....	47
Proportion .....	27	Tolerance .....	47
Pythagoras' Theorem .....	63	Trapezium .....	59
Quadrilateral		Trend.....	25
Area .....	59	Triangles.....	63
Range.....	40	Area.....	17
Rate.....	27	Units.....	5
Ratio .....	27	Volume.....	7, 17
Rectangle		litres and millilitres.....	18
Area .....	17	of a cube.....	18
Reduction .....	64	of a cuboid.....	17
Related Measurements .....	49	Volumes	
Repayments (loan) .....	38	Cylinder .....	62
Rounding .....	5, 9	Prism .....	62
Scale Drawings.....	49	Weight .....	7

All information in this revision guide has been prepared in best faith, with thorough reference to the documents provided by the SQA, including the course arrangements, course and unit support notes, exam specification, specimen question paper and unit assessments.

These notes will be updated as and when new information becomes available.

We try our hardest to ensure these notes are accurate, but despite our best efforts, mistakes sometimes appear. If you discover any mistakes in these notes, please email us at

[david@dynamicmaths.co.uk](mailto:david@dynamicmaths.co.uk).

An updated copy of the notes will be provided free of charge!

We would like to hear any suggestions you may have for improving our notes.

**This version** is version 3.0: published December 2018.

**Previous versions:**

Version 2.0: published May 2017, Version 1.2: published August 2015

Version 1.1: published December 2014, Version 1.0: published May 2014

With grateful thanks to **Arthur McLaughlin** and **John Stobo** for proof reading.